A Little Bit of Math

John McCleary

Vassar College LLI

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President Abraham Lincoln

"He studied and nearly mastered the Six-books of Euclid (geometry) since he was as member of Congress. He began a course of rigid mental discipline with the intent to improve his faculties, especially his powers of logic and language. Hence, his fondness for Euclid, which he carried with him on the circuit till he could demonstrate with ease all of the propositions in the six books; often studying far into the night, with a candle near his pillow, while his fellow-lawyers, half a dozen in a room, filled the air with interminable snoring."

from *Short Autobiography of 1860*

What is Euclid about? Why was Lincoln so interested?

The Pythagorean Theorem

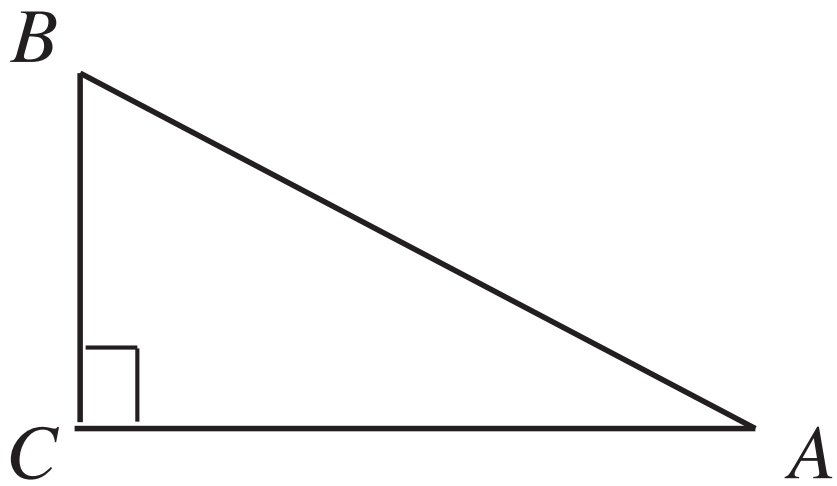
What is the Pythagorean Theorem (PT)?

$$a^2 + b^2 = c^2.$$

Such notation would only be common after the time of René Descartes (1596–1650).

The Pythagorean Theorem

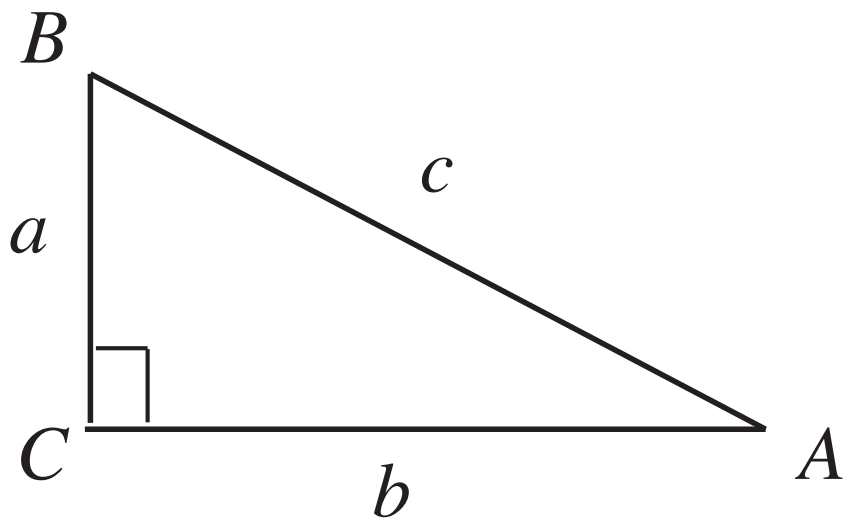
Begin with a **right triangle**.



The Pythagorean Theorem. For a right triangle $\triangle ABC$ with right angle at C, the squares on the legs sum to the square on the hypotenuse.

The Pythagorean Theorem

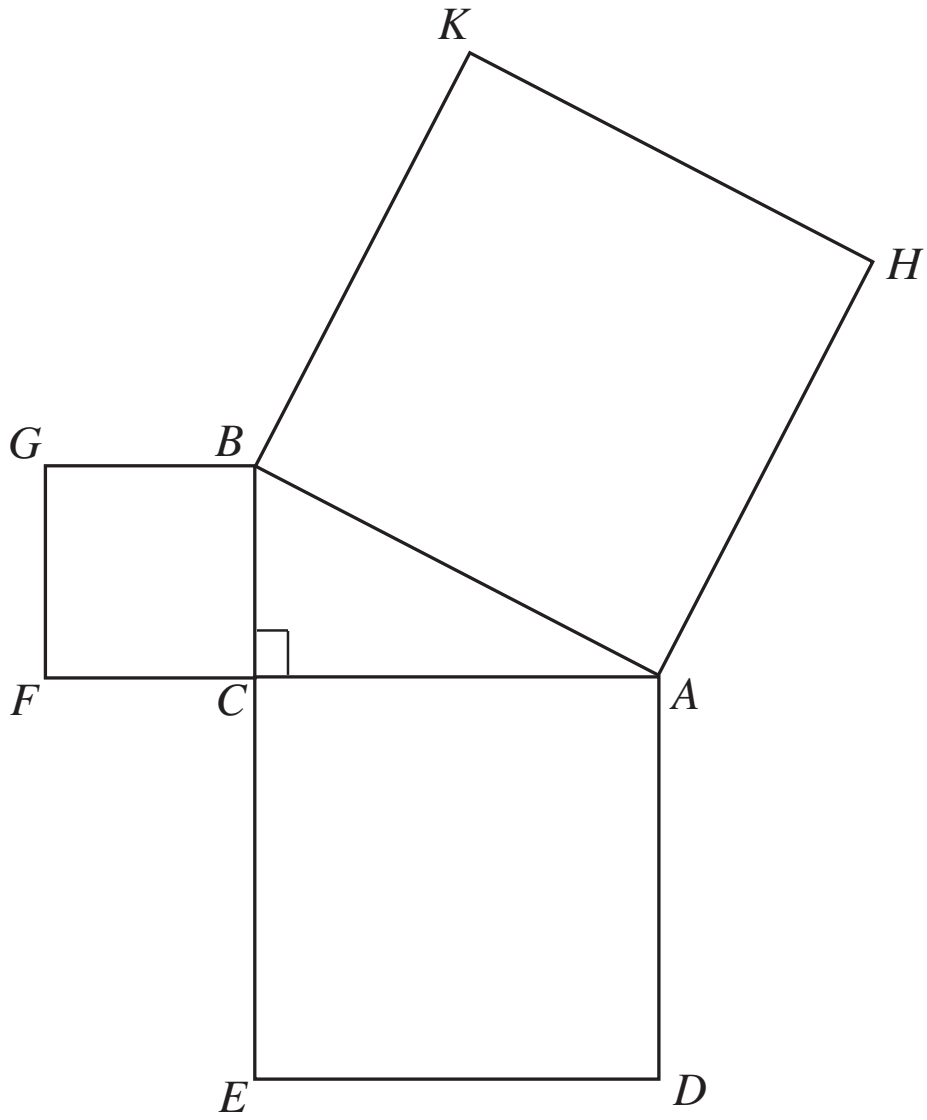
This is a matter of area. It was Leonard Euler (1707–1783) who introduced the notation for which the length of the side opposite a vertex is labeled by the lower case version of the vertex.



In area we would write $a^2 + b^2 = c^2$.

The Pythagorean Theorem

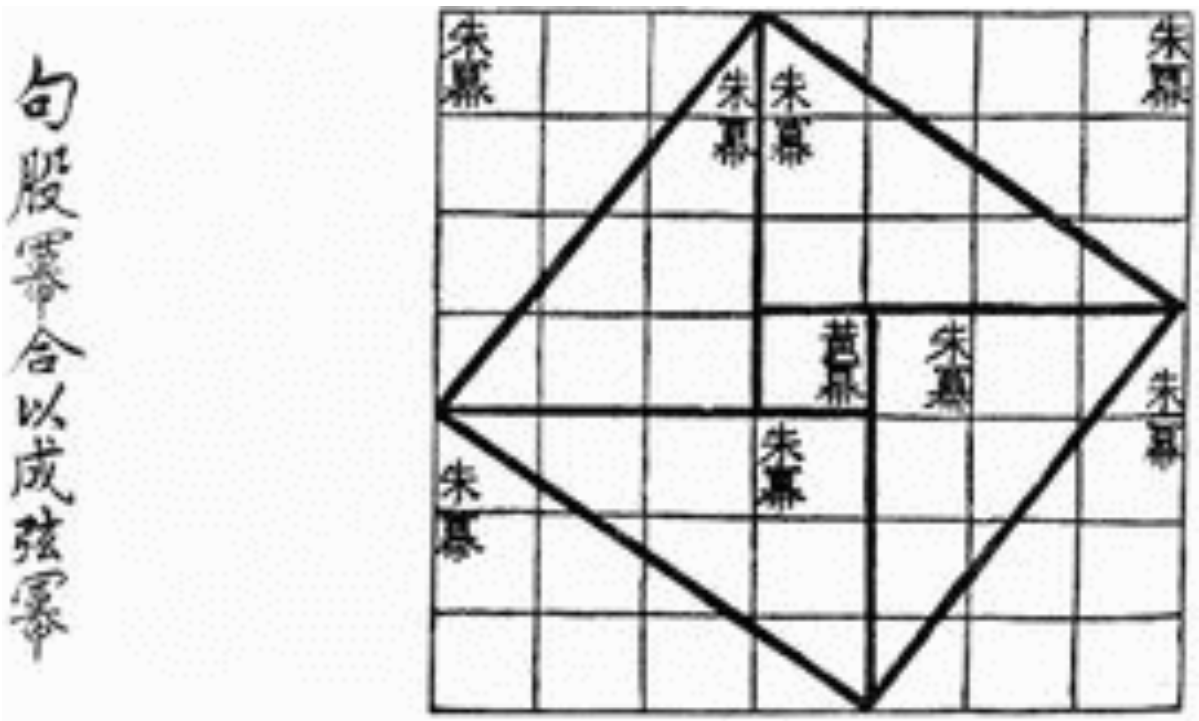
But Euclid would think:



The Pythagorean Theorem

The right-angle property was known in special cases. The Babylonians and Egyptians knew of the 3-4-5 triangle as a construction idea. They had the *rope-stretchers*.

In a Chinese classic *Chou Pei Suan Ching* (ca. 250 B.C.E.) there is a diagram that demonstrates the 3-4-5 triangle.



The Pythagorean Theorem

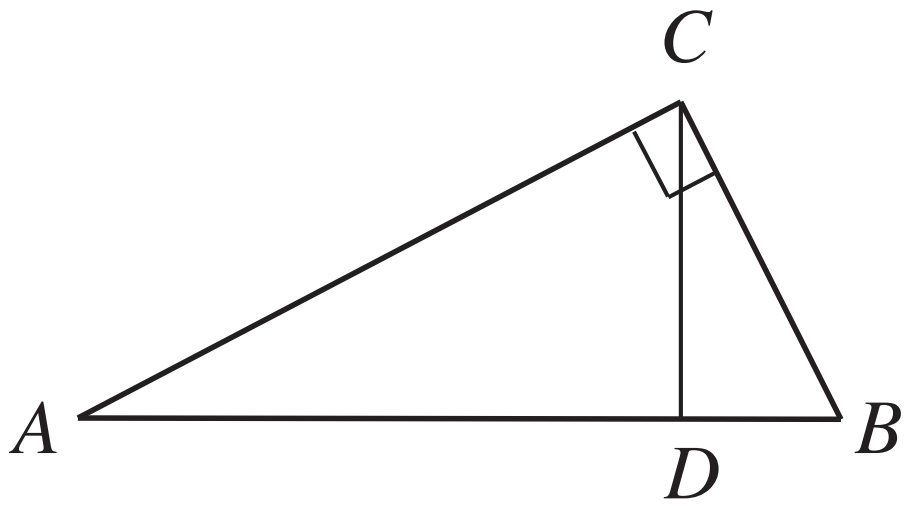
How did the Pythagoreans know the Pythagorean Theorem?

Perhaps proportionality. Recall:

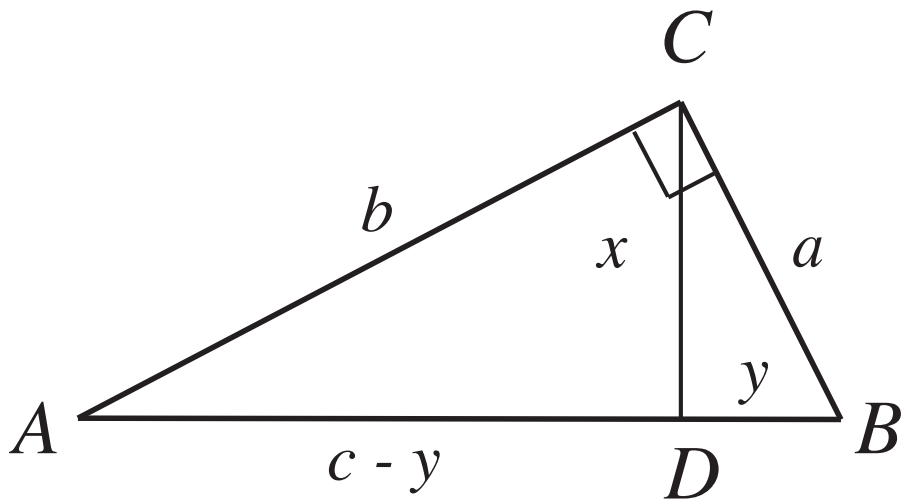
Two triangles are *similar* if they have the same interior angles.

The sum of the interior angles of a triangle is two right angles (180°)

Proof by proportionality

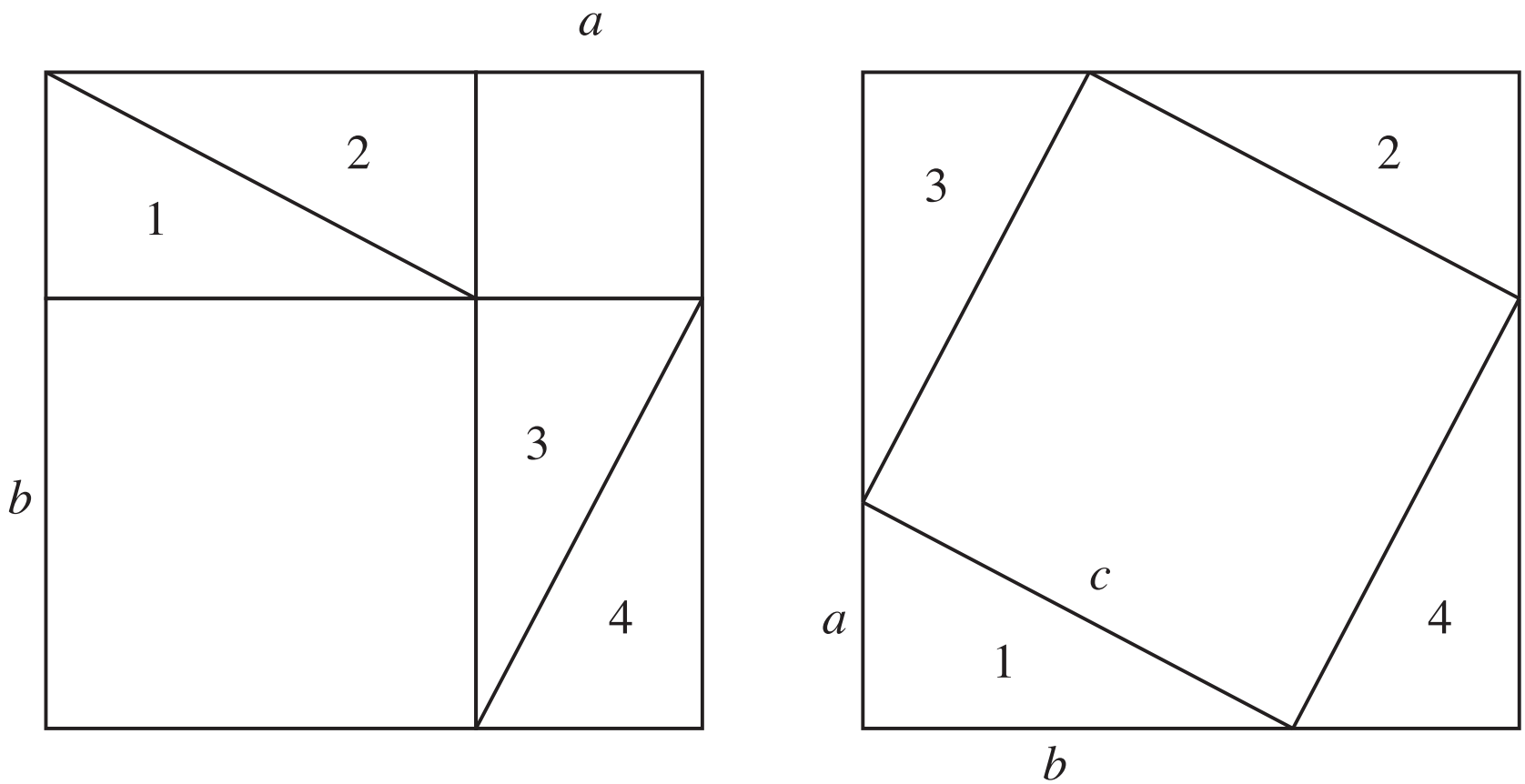


Proof by proportionality



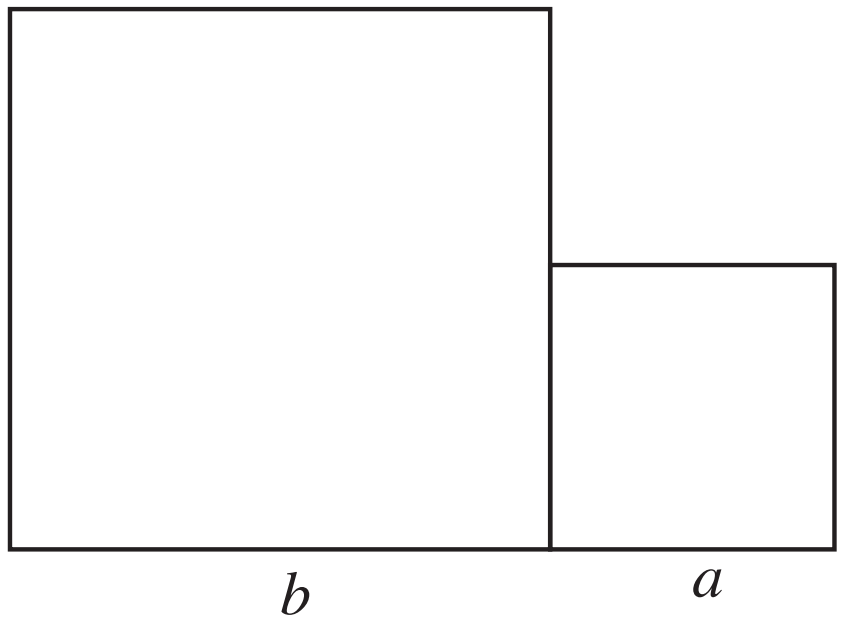
$$\frac{y}{a} = \frac{x}{b} = \frac{a}{c} \text{ and } \frac{x}{a} = \frac{c-y}{b} = \frac{b}{c}.$$

Proof by cut-and-paste

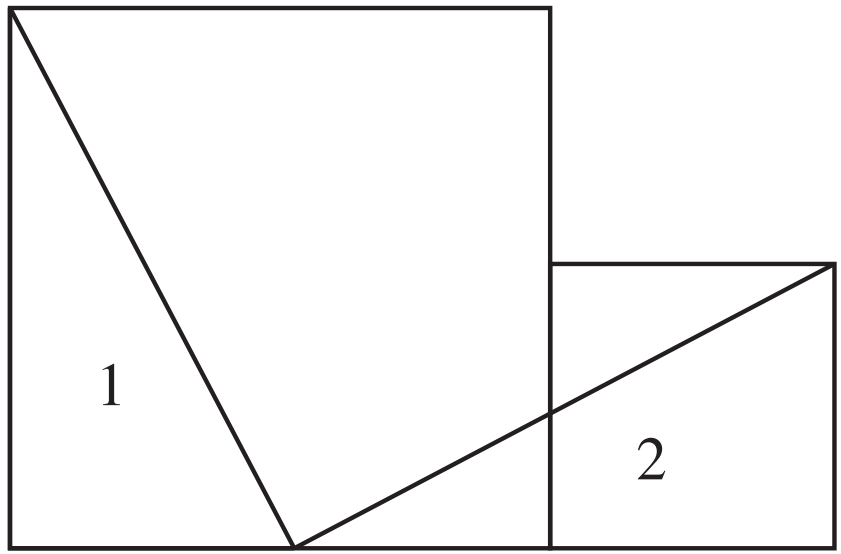


Proof by cut-and-paste

A proof due to Thabit ibn-Qurra (826–901 C.E.).



Proof by cut-and-paste



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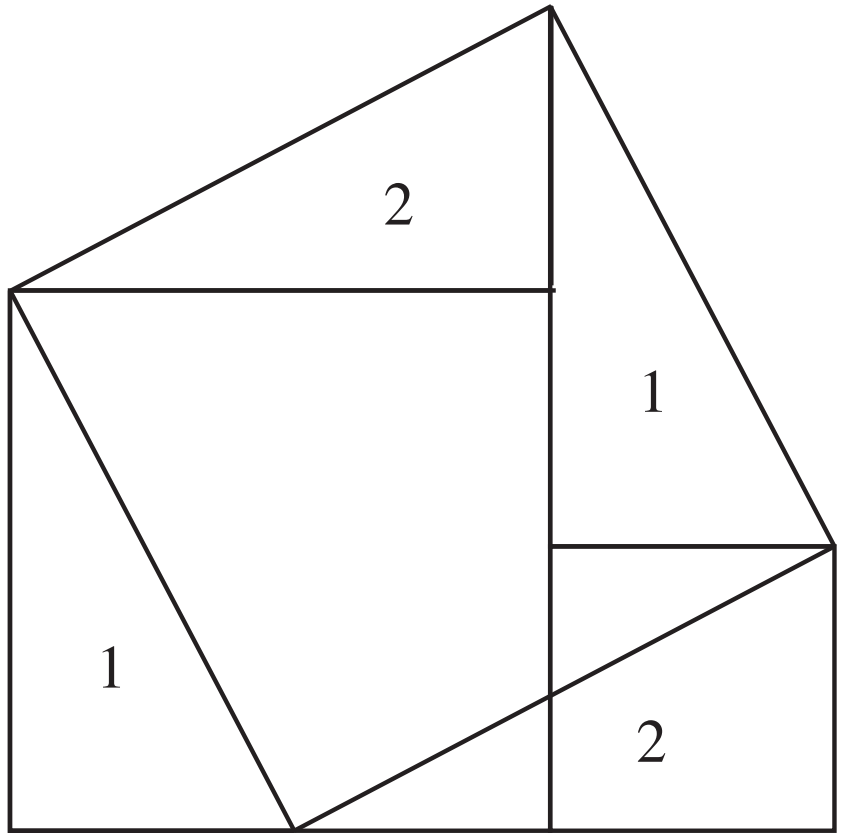

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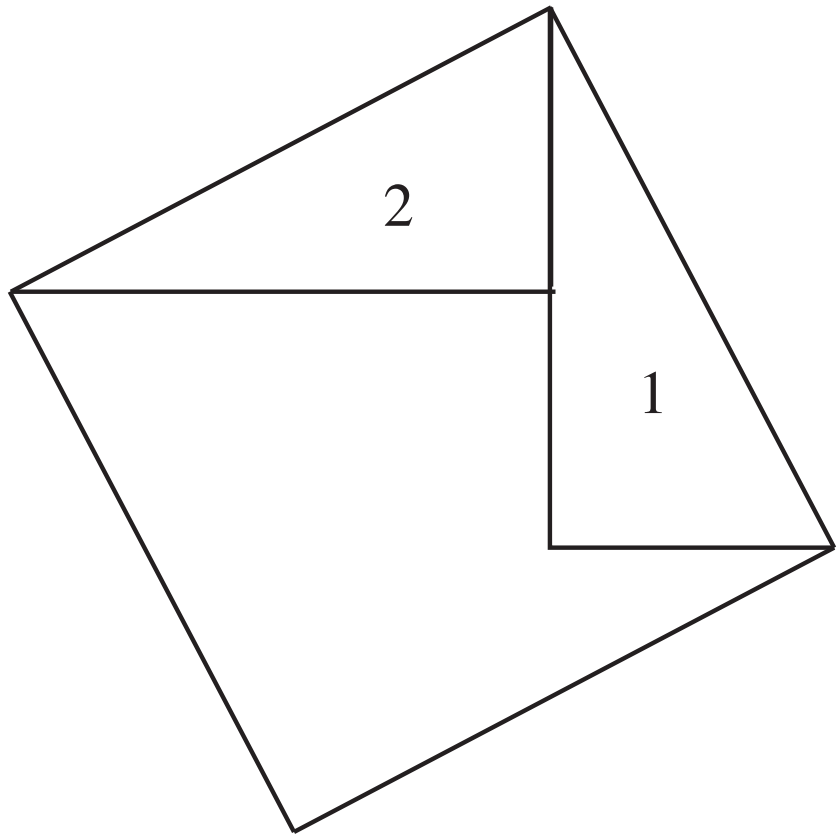
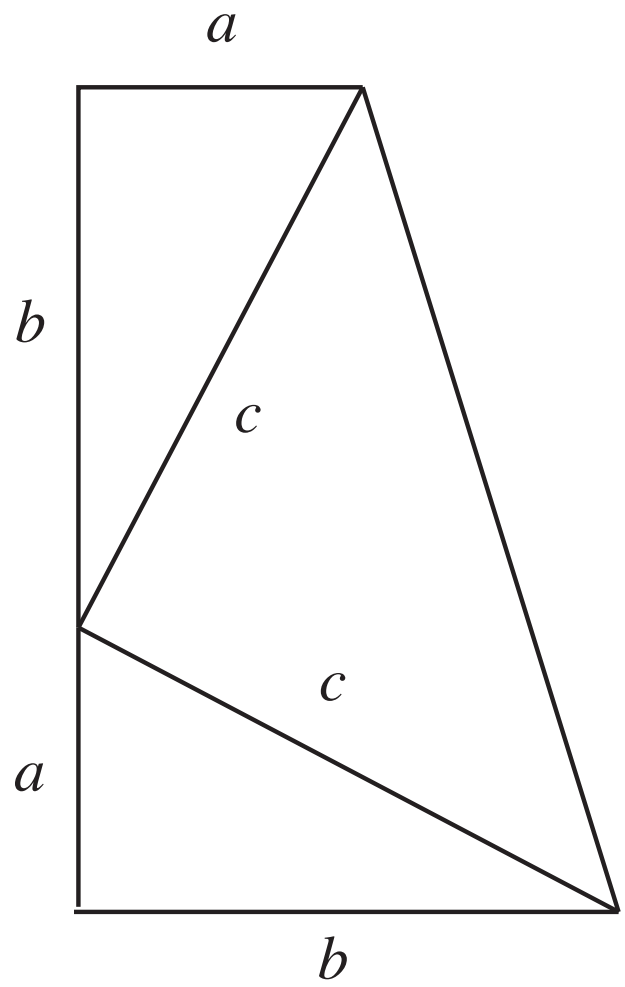
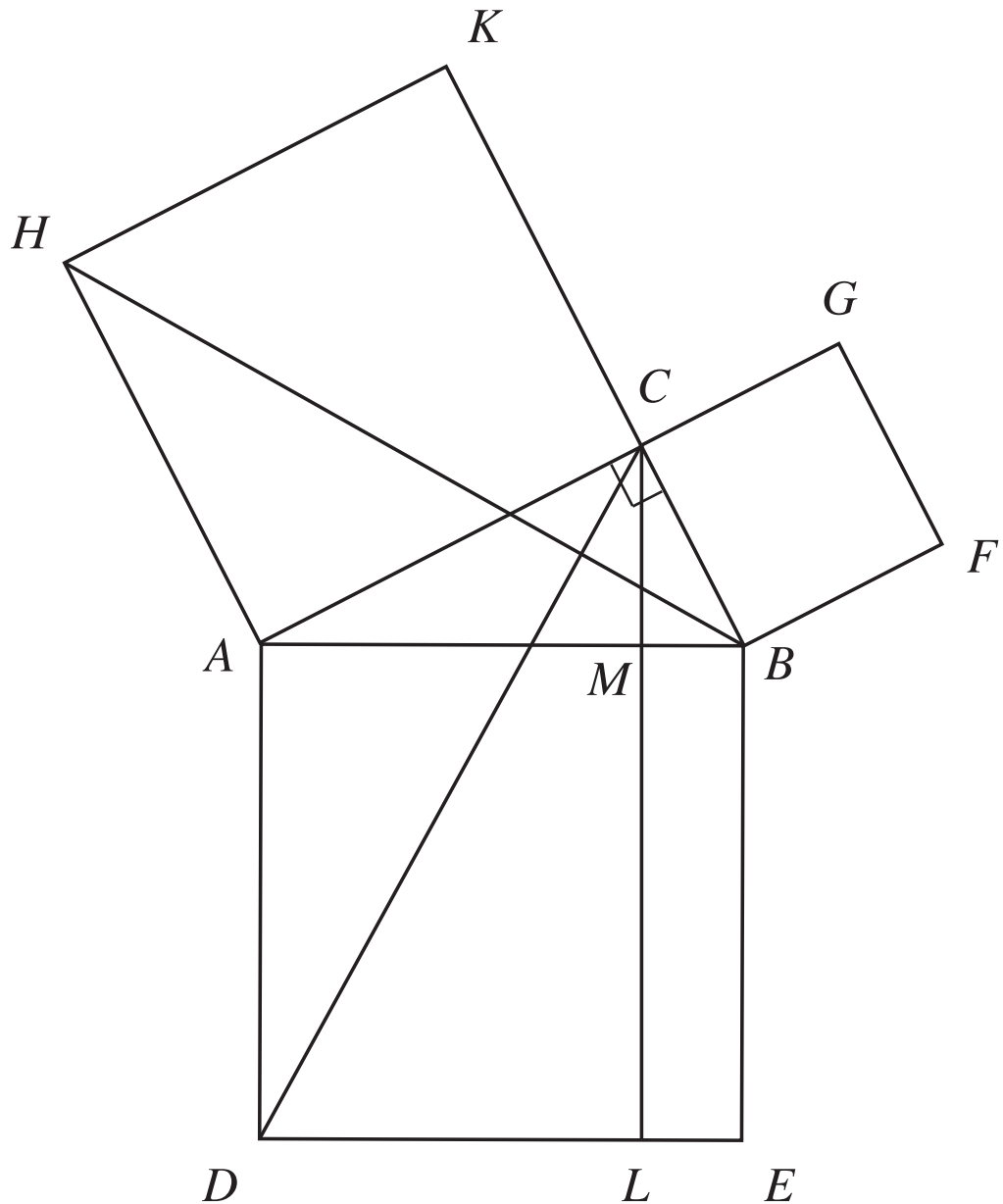


Proof by President J.A. Garfield



Proof by Euclid (Book I.47)



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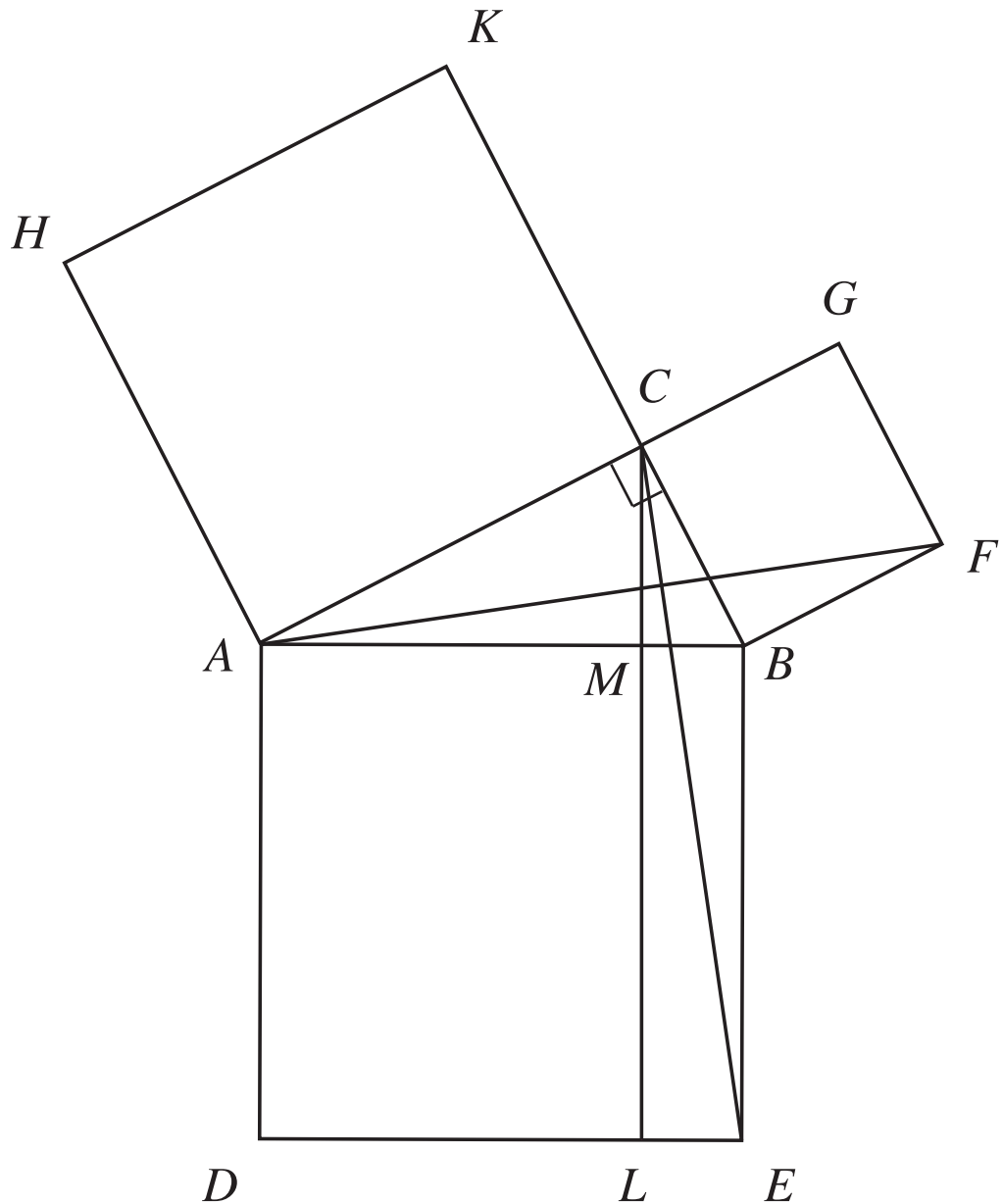

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