Parmenides 143d-144a and the Pebble-Arithmetical Representation of Number

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The purpose of this short reflection is to draw on a largely ignored, if not forgotten, ancient resource in order to suggest a fresh reading of a problematic passage in Plato's *Parmenides*, Parmenides' proof for the being of number at 143d-144a. I shall first translate the passage (section I), then note the key problems and explain why I think we should be unsatisfied with the best of the responses that, to my knowledge, have been offered in the secondary literature (section II), and then offer the fresh reading (section III). I shall close with an aside on the possible fit of our passage, under this reading, with one of the otherwise obscure teachings that Aristotle credits to Plato in his report of various unwritten teachings in *Metaphysics* A6 (section IV).

I. The passage

The passage is *Parmenides* 143d8-144a4.[1] Here is the immediate argumentative context: Parmenides is in the midst of his long and serpentine argument that "the one itself" (αὐτὸ τὸ ἕν, 143a6, 144e6) is many (144e6-7). He begins by distinguishing the characters "one," "being," and "different" (143b1-8), goes through the several ways in which any two of these can be paired and called "both" (143c1-9),[2] infers from the being of such a pair that "there are two" and that "each of these is one" (143d1-5), and infers from the possibility of "adding any one to any pair"

[1] My thanks to Rachel Kitzinger for illuminating conversation about nuances of the Greek.

[2] Throughout this passage Plato has Parmenides use the dual, referring to each of the various pairs not just as a two but, more strongly, as a couple: τινε, ἀμφοτέρω, αὐτοῖν, ἄμφω. See Brumbaugh 1961, 95; Allen 1970, 32 and 1974, 710 and 1983, 225; Curd 1990, 27.

(συντεθέντος ἑνὸς ὁποιουοῦν ᾑτινιοῦν συζυγίᾳ) that the result will be "three" (143d5-7); finally, he observes that "three is odd and two is even" (143d7-8).

With this preparation, Parmenides leads his interlocutor, the young "Aristotle,"[3] through this reflection:

> — "What about this? Given that there are two, must there not also be twice (δὶς), and given that there are three, thrice (τρὶς), since twice one belongs to two and thrice one, to three (εἴπερ ὑπάρχει τῷ τε δύο τὸ δὶς ἓν καὶ τῷ τρία τὸ τρὶς ἕν)?"
>
> — "Necessarily."
>
> — "Given that there are two and twice, must there not be two twice (δύο δὶς)? And, again, given that there are three and thrice, must there not be three thrice (τρία τρὶς)?"
>
> — "Of course."
>
> — "And what of this? Given that there are three and there are twice and given that there are two and there are thrice, must there not be both three twice (τρία δὶς) and two thrice (δύο τρὶς)?"
>
> — "Very much so."
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> — "Accordingly, there would be evens an even number of times (ἄρτια … ἀρτιάκις) and odds an odd number of times (περιττὰ περιττάκις) and evens an odd number of times (ἄρτια περιττάκις) and odds an even number of times (περιττὰ ἀρτιάκις)."
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> — "That is so."

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[3] Not Plato's great successor. Not only is the dramatic date of the dialogue about 450 b.c.e., but this *dramatis persona* is identified as "the man who later became one of the Thirty" (127d), the pro-Spartan aristocrats who, with Lysander's support, seized control of Athens in 404-403.

— “Then if that is so, do you think there is any number left over (τινὰ ἀριθμὸν ὑπολείπεσθαι), which must not necessarily be?”

— “In no way at all.” (Οὐδαμῶς γε.)

II. Problems, Familiar Responses, Dissatisfactions

There are at least three conspicuous apparent problems[4] with Parmenides’ four-fold classification of number.

[1] *The first problem: the apparent omission of the primes.* By his leading question at 144a2-3, Parmenides wins Aristotle’s emphatic agreement that “there is [no] number left over.” But Parmenides’ four classes, evens an even number of times, odds an odd number of times, evens an odd number of times, and odds an even number of times, do not include prime numbers.[5] For each prime has as its only factors itself and one, and one was held to be neither odd nor even[6] — or, perhaps,[7] both odd and even at once.[8] If one is neither odd nor

[4] These three problems are succinctly stated by Allen 1974, 713n45 and 1983, 311n159.

[5] This objection was made as early as Aristotle, at least on the widely accepted reading of ἔξω τῶν πρώτων at *Metaphysics* A6: 987b34. I shall offer a possible alternative to this reading in section IV below.

[6] The exceptional status of one in ancient Greek arithmetic is widely acknowledged. If number is defined as “a multitude of units” (see Euclid, *Elements* VII, Def.2), then one is not a number; rather, as Aristotle, for whom the notion of a multitude of countable units figures in all of his definitions of number (see Heath 1921, 70), writes, one is “the measure” (τὸ μέτρον) and the “starting-point” (or “principle,” ἀρχή) of number (*Metaphysics* N1: 1088a5-7) and, so, stands apart from and is prior to the rest of the series of integers. See n8 below.

[7] Philolaus fr. 5, discussed in M.E. Hager 1962, 1-2.

[8] That Plato grants to his *dramatis persona* Parmenides the recognition that one ought not be classified as odd or even is evidenced by his having Parmenides jump over one in silence and

even, then of course it cannot count as a factor in any of the four classes. If, on the other hand, it is considered both odd and even, then each of the primes, as a combination of an odd, namely, itself, and a both-even-and-odd, namely, one, will no more fall within the class of odds an odd number of times than within the class of odds an even number of times (or, if for the moment we take this as its equivalent, evens an odd number of times), in which case it would no more count as a prime than as not a prime.

Are there resources in our text to overcome this objection? At least three main possibilities have been advanced. We may reach back to the earlier argumentation and cite 143d5-7 as a precedent for supplementing multiplication with addition: there, as we have noted, Parmenides argues that "adding any one to any pair" will yield three; if we take "any pair" as an exemplary case of even and three as an exemplary prime, we can take this addition of one to two as a precedent licensing the account of each of the following primes as the result of adding one to the even number that immediately precedes it.[9] Or, secondly, we

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declare two to be the first even number and three to be the first odd number (143d7-8). — It is also striking that in the *Phaedo* when Socrates needs a paradigmatic participant in the form Oddness, Plato has him cite three, not one. The only text that appears to differ from the *Parmenides* and the *Phaedo* on this question is *Hippias Major* 302a. As Knorr 1975, 167 writes, "… the *Hippias*-passage utilizes a conception of the unit (as an odd number) not recognized in the arithmetic theories known to Plato, Archytas, and Philolaus." I am persuaded by Allen's observation 1974, 711n37 that Plato there has Socrates "treat one as an odd number" as a "dialectical" tactic, not "a matter of abstract number theory."

[9] This appears to be the approach of Cornford 1939, 141n2 (for objections to his formulations, see Allen 1974, 713n44 and 1983, 310n158), of Brumbaugh 1961, 98 ("The primes, omitted in the cross-classification [of odds and evens], are presumably generated by addition."), and of Sayre 1996, 170-171. Scolnicov 2003 dissents; arguing that Plato conceives number "primarily as proportions and *structures*, and only secondarily as (denumerable) collections" (106), he denies

can dispense with the citing of any prior passage and argue, instead, that multiplication is a form of addition so that when Parmenides distinguishes his four putative multiplication sets, he tacitly invokes addition and invites us to add one wherever it might be needed.[10] Or, thirdly, we can point ahead to Parmenides' inference at 144a5-7 that "given that there is number, there would be many, namely, an unlimited number of beings" and, reading this as a

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that "primes greater than three" can be established "by the addition of a unit to a previously derived collection, for this would make them συνθέσεις μονάδων" (105). Scolnicov's interpretation of Plato's conception of number is, in my view (Miller 1999, 76-83), a good insight into the notion of number as it emerges in the final four members of the five mathematical studies that Plato has Socrates prescribe for the would-be philosopher in *Republic* VII, for in the several forms of geometry and in harmonic theory number always appears in and as ratio, even when, as with the paradigm case of the relation of the side and the diagonal of the square, there is no common unit of measure; but as I shall argue, it misses the notion of number that governs the first study, "logistic and arithmetic" (525a, also 522c), and that is the conception in force in *Parmenides* 143d-144a (see section III below). An outlier is Turnbull 1998, whose ingenious notions of the "two machine" and the "three machine" appear, if I follow his constructions, to derive from Parmenides' pairs and triads at 143c-d operations of selective grouping that, in opportune combinations, can yield prime numbers of the things grouped; as he acknowledges, however, "[t]here is nothing in [143a-144e] or elsewhere in the *Parmenides* to back up [this] procedure for generating the primes" (79).

[10] Allen, who at several points describes Parmenides as "proceeding by addition and multiplication" (1974, 712 and 1983, 227), also takes the stronger position that "multiplication *is* abbreviated addition" (1974, 713n44 and 1983, 310n158, my stress).

recognition of the infinite series of integers, argue that primes are tacitly established and acknowledged just insofar as they fall within that series.[11]

Each of these three responses to the question of the apparent omission of the primes can be argued and, as a supplement to Parmenides' argument at 143d8-144a4, is potentially persuasive. The problem is that Plato does not have Parmenides suggest any of these moves; on the contrary, once Parmenides has established the being of one and two and three, he proceeds straightaway to establish the existence of the four classes, and it is with reference to these — one, two, and three and the four classes — that he wins from Aristotle the agreement that "there is [no] number left over." Plato does not have Parmenides give the slightest indication that there is a problem with the status of primes. The three responses to the problem, accordingly, have the character of uninvited and *ad hoc* efforts to fill in a lacuna in his four classes that he does not acknowledge. Is there a better response, one that makes good sense of the fact that Parmenides does not acknowledge the problem in the first place?

The second and third problems: [2] *the redundancy of two of the classes and* [3] *ambiguities in class membership.* Two other problems have been much less frequently noted.[12] First, there is the striking apparent redundancy of the classes of "evens an odd number of times" and "odds an even number of times" and, in the paradigm cases by which Parmenides first establishes these, of "two thrice" and "three twice". This redundancy does

[11] This is strongly implied by Allen when he writes that "the existence of any number implies the existence of every number" (1974, 714) — but note Sayre's observant reservation at 1996, 343n17 — and, again, when he writes that "… any of Parmenides' methods — multiplication of even numbers, of odd numbers, or of odd and even numbers — will suffice to prove the existence of any number, since any of these methods, by repeated application, will prove the existence of a number larger than any number desired" (1983, 228).

[12] See the citations of Allen 1974 and 1983 in n4 above, also Brumbaugh 1961, 98, Turnbull 1998, 78, and Sanday 2014, 204n14.

not result in any number being "left over," of course; but it is a puzzling feature of Parmenides' argument, and it is even more puzzling that he does not even acknowledge it, much less offer any explanation. Secondly, there is the related problem of an apparently superfluous multiplicity in class membership: each number that is a product of an odd and an even factor — in the case at hand, six — evidently belongs to both of the two classes. This multiplicity in class membership is compounded in the case of numbers that are products of even factors as well as of even and odd factors: twelve, to take the first example of this in the series of integers, belongs at once to evens an even number of times (for it is six twice, or two six-times), to evens an odd number of times (for it is four thrice), and to odds an even number of times (for it is three four-times). As with the apparent redundancy of classes, this is, if an error at all, one of superfluity rather than omission; that is, no number is "left over" as a consequence of it. But, again, it is puzzling that Parmenides would leave his classification of numbers open to these problems without the least acknowledgment. Is there a reading of Parmenides' argument that would obviate these problems?

III. <u>A Possible Resource for Parmenides: The 'Pebble Arithmetic' of Squares and Oblongs</u>

To begin with the second problem, the apparent redundancy of the classes of "evens an odd number of times" and "odds an even number of times," we have so far left unchallenged a point on which there is a general consensus in the literature we have cited, that Parmenides' four classes are pairs of factors in straightforward relations of multiplication. But in fact this is problematic. Any such relations are commutative, and, further, each yields a product. Not only does Parmenides distinguish his third and fourth classes, "evens an odd number of times" (ἄρτια περιττάκις) and "odds an even number of times" (περιττὰ ἀρτιάκις), by the sequence of their factors, but in first distinguishing their paradigm cases, "two thrice" and "three twice" — and, as well, the cases "two twice" and "three thrice," he makes no mention at all of any products. If we seek another way to understand "evens an odd number of times" and "odds an even number of times," a way in which the sequence matters and what products they might have is, if a concern at all, not a primary one, we might turn to the so-called ‘pebble arithmetic’ of the fifth century Pythagoreans, in which numbers were represented and studied not as mere multitudes of units but, rather, as figured arrays.[13] Here the sequence of factors — if, indeed, the word “factors” is not too suggestive of the operations of multiplication to do justice to the terms involved[14] — makes all the difference. “Three twice” and “two thrice” are expressed by different figures, “three twice” by two horizontal rows of three units, an oblong rectangle that is wider than it is tall by one, namely,

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and “two thrice” by three horizontal rows of two units, an oblong rectangle that is taller than it is wide by one, namely,

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[13] For an illuminating discussion of the ‘pebble arithmetic’ and its mathematical power, see Knorr 1975, ch. V.

[14] This is also, arguably, a problem with the familiar translation of the suffix -άκις as “times” in ἀρτιάκις (“even times”) at 143e7 and ff. and in περιττάκις (“odd times”) at 144a1 and ff. To the English ear this suggests multiplication, and while this meaning is quite possible, it is too specific to stand without contextual support; -άκις need signify no more than ‘[taken] a number of times,’ without respect to the multiplication of a number by another number and the product this yields. Hence my more neutral translations of ἀρτιάκις as “an even number of times” and of περιττάκις as “an odd number of times” in Section I. See LSJ under ἀρτιάκις and περιττάκις.

Thus “three twice” and “two thrice” — and, correspondingly, “odds an even number of times” and “evens an odd number of times” — would no longer be redundant.[15]

Why, however, would Plato be interested in this so-called ‘pebble arithmetic’? To step back and begin with a general reflection, the possibility that Plato might be making use of the pebble arithmetic shouldn’t come as a surprise. In his reconstruction of the history of fifth and fourth century Greek mathematics, Wilbur Knorr has argued that the representation of number by means of figurative arrays of discrete units — triangular and square and oblong rectangular arrays of, e.g., dots or pebbles — was still practiced in the early Academy and, indeed, was preserved as a mode of arithmetic even while “mathematicians like Theaetetus” were also anticipating Euclid by “formaliz[ing] parts of the theory of number by … represent[ing number] by continuous quantities, [namely,] lines, plane figures and solids ….”[16] Indeed, the co-presence of these two modes of representation, argued for as an historical matter by Knorr, appears to have been a constitutive feature of the pedagogical system by which Plato has Socrates order the five disciplines in his prescription of mathematics as a propaedeutic for the philosopher-to-be in *Republic* VII. A full recapitulation of this ordering and its significance would go beyond the bounds of our present reflection and must be reserved for another occasion.[17] For the present it should suffice to gather the following three observations: first, that Socrates asserts that the first of the five studies, “logistic and arithmetic,” takes as its subject matter an indefinite plurality of absolutely equal, invariant, and partless units (*Republic* 526a); second, that, according to

[15] Brumbaugh 1961, 98 and Sanday 2014, 204n14, cited in n12, notice this, but neither goes on to work out the implications of the possibility that Plato may be making use of the pebble arithmetical representation of numbers for our other two problems, [1] and [3].

[16] Knorr 1975, 145-146. Indirect evidence of the preservation of pebble arithmetic in the Academy is Speusippus’ tract “On Pythagorean Numbers.”

[17] I have attempted a recapitulation of this ordering and a study of its significance on several other occasions. See Miller 1999, also Miller 2007, 318-323.

Socrates, even while the arithmetician knows that these units are purely intelligible, he nonetheless "makes use of visible figures" (τοῖς ὁρωμένοις εἴδεσι προσχρῶνται, 510d5) to represent them; and third, that, albeit with a degree of pedagogical irony, Socrates repeatedly encourages Glaucon to see in calculation and arithmetic an indispensable help for the warrior in determining the spatial "arrangements" of his troops (τὰς τάξεις, 522d3, ταξέων, e4, διὰ τὰς τάξεις, 525b4) — that is, in designing the appropriate "formations" of his soldiers on the battlefield and on the march and in setting up camp (526d2-5, esp. ὅσα δὴ ἄλλα σχηματίζουσι τὰ στρατόπεδα). If, taking these three observations together, we step back to ask what "visible forms" for the representation of the homogeneous and partless units of arithmetic would fit with designing the "arrangements" and "formations" of troops, we should be led to think of the triangular and square and oblong arrays of the pebble arithmetic. And if that is right, then Plato has Socrates preserve the two modes of the representation of number — that of arrays of units and that of lines and planes and solids — as the first and the middle three, respectively, of the five mathematical disciplines; it would be part of the pedagogical process of the five studies that the student moves from the representation of numbers by figured arrays of units in logistic and arithmetic to the representation of them by the figures of plane and solid geometry and of theoretical astronomy.[18]

Recognizing the possible place of the pebble arithmetic in the practice of mathematics in the Academy and in the pedagogical curriculum of the *Republic* should motivate us to consider its possible presence in the *Parmenides*. But what difference might its presence there make? We

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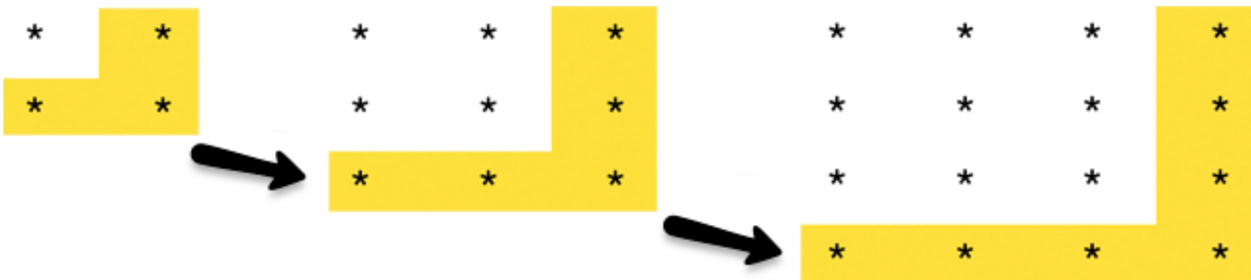
[18] In short, the student moves from logistic~~s~~ and arithmetic to the three geometrical studies by dropping the units in the figurative arrays of logistic~~s~~ and arithmetic and focusing his attention on the figures, as such, that the arrays of units express; analogously, he moves from the three geometrical studies to harmonics by dropping the figures and focusing his attention on the ratios, as such, that the figures express. On the pedagogical value of these refocusings for the student preparing for dialectic, see the work cited in n17.

have just seen how recourse to the pebble arithmetic eliminates the problem of the redundancy of Parmenides' classes. It also eliminates both the problem of the apparent omission of the primes and the problem of ambiguous class membership. To see this, consider the pebble arithmetical procedures for establishing the existence of the series of odd numbers and the existence of the series of even numbers by expanding square and oblong numbers, respectively.[19]

The square numbers take as their starting-point the unit, * , with the first square number constructed by adding a new unit to the right (to form a horizontal row of two) and another new unit directly below (to form a vertical column of two) and, to complete the square, a third new unit in the open space below the first of the two added units and to the right of the second of them, like so:

[19] I owe a great debt to a fine paper, "Revisiting Plato's Generation of Number (*Parmenides* 143c1-144a5)," prepared for an SAGP meeting in 2008 by Michael Barkasi, then an undergraduate at Kutztown who later went on to receive his doctorat in philosophy at Rice. My own earlier work on the *Parmenides* (Miller 1986 [1991], 1995) and on the pebble arithmetical representation of numbers in the first of the five studies in *Republic* VII (Miller 1999) moved me to seek him out when I first came across an announcement of his paper, and while he was writing it, we shared a rich correspondence from which I benefited greatly. Unfortunately, he did not go on to publish his paper, so one of the services of the present essay, above all in Section III, is to make available, with his advance approval but in my own more informal manner of presentation, much of what we shared in the course of our correspondence.

Thus we have "two twice," the first member of the class of "evens an even number of times." The three added units, forming a right angle around the first unit, make up the first "gnomon."[20] Each successive expansion of the square is achieved by adding a new gnomon, like so[21]



and so on.[22] This form of representation makes intuitively evident how the successive expansions of the square alternate between an "even an even number of times" and an "odd an odd number of times" and give us, in the series of gnomons, the series of odd numbers: starting

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20 The original referent of the Greek γνώμων was a carpenter's square, the tool by which the carpenter marked a right angle; Knorr 1975 gives its mathematical sense as "that number which, when added to a term in a given class of consecutive figured numbers, produces the next term in that class" (143). I have indicated each of the gnomons in my diagrams by highlighting the asterisks, or 'ones,' that constitute them.

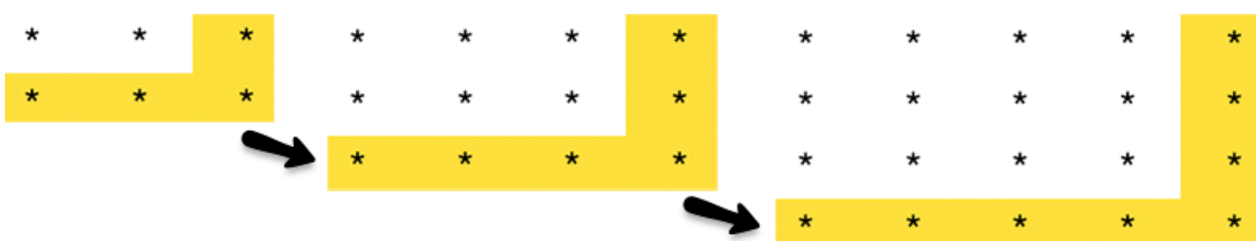
21 My thanks to Glenn Johnson of Penn State for help with the design of this and the following diagrams.

22 An alternative but equivalent mode of representation is to "wrap" each new gnomon "around" (cf. περιτιθεμένων, Aristotle *Physics* 203a13) the preceding figure, setting the pebbles that make it up on the two sides of the figure opposite to the sides on which the preceding gnomon was placed; thus the figures would be expanded in opposite directions, first downwards and to the right, then upwards and to the left, then downwards and to the right, then upwards and to the left, and so on. I have chosen the mode of expanding each figure in the same two directions simply for convenience's sake.

with the three in the gnomon of the square that is "two twice," we move to the five in the gnomon of the square that is "three thrice," then to the seven in the gnomon of the square that is "four four-times," and so on.

The oblong numbers, in turn, take as their starting-point a dyad, * * , with the first oblong number constructed by adding a new unit to the right (to form a row of three), a new unit below each of the initial two (to form two columns of two), and, to complete the oblong, a fourth new unit in the open space below the first added unit and to the right of the second two added units, like so:

Thus we have "three twice," the first member of the class of "odds an even number of times." The four added units, forming a right angle around the initial two, make up the first gnomon, and each successive expansion of the oblong is achieved by adding a new gnomon, like so



and so on. And again, as with the expansion of the square, so here, it is intuitively evident how the successive expansions of the oblong alternate, now between an "odd an even number of times" and an "even an odd number of times," and give us, in the series of gnomons, the series of even numbers: starting with the four in the gnomon of the oblong that is "three twice," we move on, preserving the shape of the oblong[23] in each expansion, to the six in the gnomon of the oblong that is "four thrice," then to the eight in the gnomon of the oblong that is "five four-times," and so on.

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23 We must add the obvious caveat that in the series of oblongs the preservation of the shape is inexact: since one dimension (in our diagrams, the width) of the oblong exceeds the other (the

And of course — setting in abeyance the question of the status of the first unit and the first dyad, the one and the two, that are the starting-points for the two series — the series of the odds presented in the series of the gnomons in the successive expansions of the square and the series of the evens presented in the series of the gnomons in the successive expansions of the oblong are together exhaustive of all number; there is no "number left over."

Surveying these two sets of figures, we can see immediately how reading Parmenides' four classes of numbers as classes of pebble arithmetical figurative arrays obviates each of the three problems that otherwise seem to burden his proof for the being of number. Rather than being omitted, the primes arise unproblematically within the series of odd gnomons. Nor is there any issue of ambiguity in class membership, for each member of each of Parmenides' four classes is represented by just one pairing of equal or non-equal rows and columns of units. And, to repeat what we already established in our opening observation in this section, the two classes, "evens an odd number of times" and "odds an even number of times," are not redundant; each of these phrases signifies a single figure, and the two figures differ.[24] No wonder, then, that Parmenides

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height) by one, preserving the oblong as we expand it involves a regular diminishing of the ratio of that excess; we move from a "three twice" to a "four thrice" to a "five four-times," etc.

[24] This is not to deny, of course, that we mightn't equally well represent the first oblong number as "three twice" (as we in fact did), hence as a member of "odds an even number of times," or as "two thrice" (as we might have but did not), hence as a member of "even an odd number of times." But whichever way we choose to begin, the oblong produced by the next expansion will be a case of the reverse pairing, and the series of expansions will proceed as an alternation of the two classes. If we take our first pairing to be "three twice," that is, a member of "odds an even number of times," its first expansion will yield its reversal, a member of "evens an odd number of times," and the expansion of that member of "evens an odd number of times" will yield *its* reversal, a member of "odds an even number of times." And the analogous alternation will result

fails even to acknowledge any of the three problems; if, that is, it is right that he is drawing on the pebble arithmetical expansions of the square and the oblong and is focused on the two series of gnomons they yield, none of the three problems arise at all.

IV. Postscript: a possible fit with Aristotle's report in *Metaphysics* A6 of Plato's derivation of numbers?

In *Metaphysics* A6 Aristotle claims that Plato held that the One (τὸ ἕν) and the dyad of the Great and the Small (τὸ μέγα καὶ τὸ μικρόν) are jointly the ultimate causes of number.[25] And he reports specifically that "[Plato] made the other nature [than the One] a dyad because of the ready derivability (τὸ … εὐφυῶς … γεννᾶσθαι) of the numbers (except for the primary ones [ἔξω τῶν πρώτων]) from it, as though from something malleable (ὥσπερ ἔκ τινος ἐκμαγείου)" (987b32-988a1). Though we read *Parmenides* 143d-144a as offering a proof for the being of number, not an account of their formation or genesis, our suggestion that he is drawing on the resources of the pebble arithmetic does invite one to bring *Parmenides* 143d-144a into relation with what Aristotle reports in this cryptic sentence. Have we happened upon the process — or,

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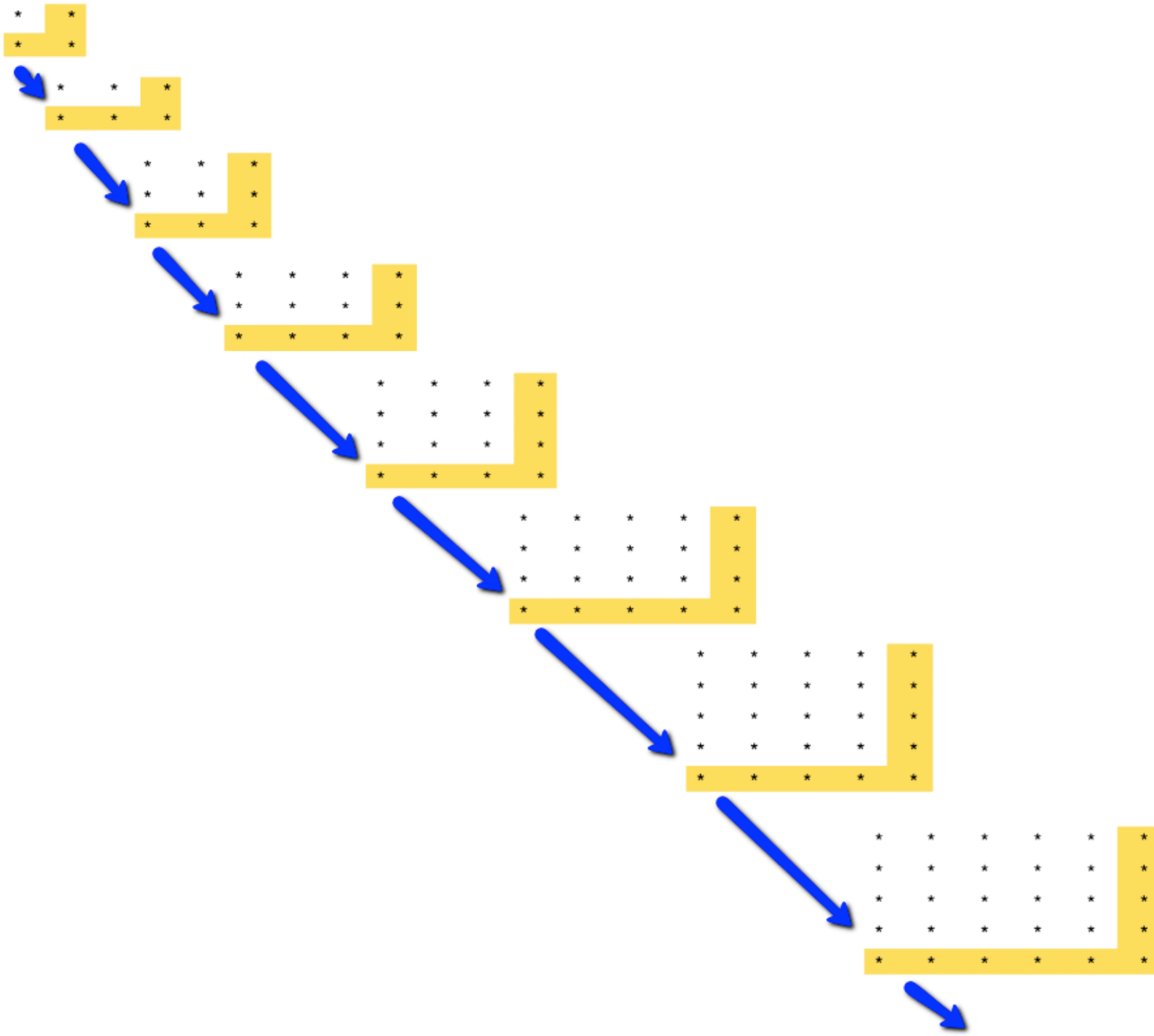
if we begin, instead, with "two thrice," that is, with a member of "evens an odd number of times," and expand it to yield a member of "odds an even number of times," and so on.

25 In the following pages I restrict myself to the question of the process of deriving the numbers from the Great and the Small as Aristotle raises it at *Met.* A6: 987b32-988a1; I have treated the distinct question of the status of the One and the Great and the Small as the joint causes of the being of the series of integers, as this is suggested by *Parmenides* 144b-e, in Miller 1995, 612-614. For the all-important distinction between Aristotle's report that the One and the dyad are responsible for the being of the series of integers and the (supposed — but, I have argued, widely misinterpreted) report that they are responsible for the being of the forms, see Miller 1995, 599-600 and 622-629.

more precisely, the structures that serve and illustrate each of the stages of the process[26] — of the "derivation" of number that Aristotle credits to Plato? In section III we glimpsed as two processes — the expansion of pebble-arithmetical squares and the expansion of pebble-arithmetical oblongs — an in-principle endless derivation, in the gnomons by which each expansion is achieved, of the series of odds and the series of evens, respectively. Suppose, now, we pair and alternate the stages of these series, moving from the first "even an even number of times" square to the first "odd an even number of times" oblong to the first "odd an odd number of times" square to the first "even an odd number of times" oblong and then moving through this whole series in the same order again and again, in principle forever? As is easily seen by re-ordering and then extending the series of squares and oblongs in our two main diagrams in section III, these stages would be the figures for "two twice," "three twice," "three thrice," "four thrice," and then "four four-times," "five four-times," "five five-times," "six five-times," and so on, and the series of gnomons by which these expansions would be achieved would give us, correspondingly, the numbers three, four, five, six, seven, eight, nine, ten, and so on.

[26] It is important to stress that by the reflections in this section I do not mean to imply that in *Parmenides* 143d-144a Plato has Parmenides offer a deduction or derivation of the being of number; Parmenides' argument is, as Allen 1970, 1974, 712-714, and 1983, 227-228 and Schofield 1972, 103, rightly stress, an existence proof, not a deduction or derivation from prior principles. But this does not prevent us from recognizing, in Parmenides' four classes, the conceptual terms by which, when their members are set in the right order and taken as stages of an endlessly iterable sequence, the series of integers might be derived.



On three counts, our reading of *Parmenides* 143d-144a seems to provide the materials, so to speak, for an interpretation of the thought that Aristotle credits to Plato at *Metaphysics* A6: 987b32-988a1. (i) As we have just shown, by pairing and alternating our expansions of the pebble-arithmetical squares and oblongs, one can "readily derive," at least in principle, all the numbers in their proper order — with the exception, to be considered shortly, of "one" and "two." (ii) If we think of each of the instantiations of the eidetic dyad of the Great and the Small as, albeit in a conceptual-imaginative rather than a sense-perceptible sense, a space-taking magnitude, the "pebble arithmetic," in representing numbers not just as multitudes but rather as figurative arrays of units, provides us with just the sort of magnitudes that the instantiation of the Great and the Small requires. Actual arrays of actual pebbles (or, needless to say, of actual asterisks or dots, as in our figures in section III) do, of course, take up actual spreads of physical space. But the units these pebbles represent are, as Socrates stresses in *Republic* VII: 526a, absolutely equal, invariant, and partless, and, so, they are not physical and not sense-perceptible; rather, they exist as the first class of the much disputed "mathematicals" and dwell, so to speak, as "intermediates between" sensibles and forms, as Aristotle reports at *Metaphysics* A6: 987b14-18.[27] (iii) Further, if we focus on the alterations of these arrays as successive shapings by which the conceptual-imaginative space that each array takes is reshaped to constitute the next, we touch on the dynamism that makes appropriate the simile of the Great and the Small as an ἐκμαγείόν τι, a "something malleable" — and further, now to exploit the prefix ἐκ-, a "something malleable out of which" something else, namely, the series of numbers, might be fashioned.[28]

To these three counts, a fourth, already briefly alluded to in n5 above, might now possibly be added. We have argued that our pebble-arithmetical reading of Parmenides' four classes prevents the apparent problem of the omission of the primes from arising in the first place; the primes present themselves unproblematically within the series of square numbers as the arrays of units that make up various of its gnomons. But we have also seen — and now in two distinct ways, one textual and the other mathematical-figurative — that "one" and "two" have a special status in the formation of Parmenides' four classes. Textually, Parmenides is able to establish the being of his four classes only on the prior bases of the "two" that he first finds exhibited in the

[27] For Plato's own implied argument for the being of "mathematicals" of such supersensible but intermediate status, see *Republic* 532b-c as discussed by Miller 1999, 74-76 and 2007, 318-319, and cf. *Philebus* 59c as discussed by Miller 2010, 46-47 and 65-77 (esp. 70-71).

[28] Needless to say, on this interpretation of Aristotle's sentence, not only the idea of "something malleable" but also the idea of an "engendering," a γεννᾶσθαι, of the numbers must be given the non-literal sense of a metaphor for 'derivation.'

"both" and the "pair" (143d1-2) and of the "one" that he finds "each member of the pair" to be (143d2-5); for the "two" is the first of the evens, and the addition of the "one" to the "two" yields the first of the odds, and it is the various pairings of these, the even and the odd, that first yield each of the four classes. Mathematical-figuratively, the "one," that is, a single unit, is the starting-point for the formation, by the addition of the succession of gnomons, of the series of square numbers, and the "two," that is, a dyad of units, is the starting-point for the formation, by the addition of its succession of gnomons, of the series of oblong numbers. Thus "one" and "two" are basic and, so, prior to the four classes. Could it have been to these, then — the "one" and the "two" — and not to the primes that Aristotle was referring in his report that Plato thought all of the numbers ἔξω τῶν πρώτων, "except for the primary [ones]," could be "readily derived" from the Great and the Small?[29]

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[29] Does reading τῶν πρώτων as "the primary [ones]," that is, "the primary [numbers]," imply that Plato as we are interpretively reconstructing Aristotle's report of his teachings violates the principle that, as Aristotle formulates it, "one" is not a number but rather the "starting-point" (ἀρχή) and "measure" (μέτρον) of number? (Recall n6.) Our suggested reading of *Parmenides* 143d-144a puts us in position to answer in the negative. If it is right that Plato's Parmenides is drawing on the pebble arithmetical account of the expansions of the square and of the oblong, then, precisely as the bases for the very formation of these figures, "one" and "two" are not mere members of the series generated by their gnomons; rather, to repeat Aristotle's language, they are "starting-points" and "measures" of the series of numbers. To this we might add that it is striking that Plato does not have Parmenides attempt a metaphysical account of what is basic to "one" and to "two"; instead, as we saw at the outset of these reflections and have just noted again, Plato has Parmenides find the being of "two" to be exhibited by any pairing of the eidetic characters "one" and "being" and "different," and find the being of "one" to be exhibited by the members of any such "two." This leaves it open to us to ask, what might such a metaphysical account, were Plato to offer one, consist in? In the context of Aristotle's report in *Metaphysics* A6 it is of course

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tempting to return to the starting-point of this postscript and ponder the relations of "one" to "the One" (τὸ ἕν) and of "two" to the so-called "dyad." But undertaking this difficult reflection would require a wider textual basis and a deeper array of ontological possibilities than we have had occasion to consider in this essay. For a beginning, see Miller 1995.

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