

## Hassler Whitney, the Applied Mathematics Panel, and Airborne Weapons Accuracy <sup>1</sup>

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In the years leading up to World War II, Applied Mathematics was not well represented at US universities. Centers at Brown and New York University (NYU), and the teaching of Statistics at Princeton and Columbia were the exceptions. As the war approached certain scientists anticipated the need for expert technical support and some sought ways to put in place the organizational structures needed to make such support possible.

One structure, already in place since 1916, was the National Research Council (NRC), established by President Wilson, which “brought science to bear on World War I” ([5]). By 1940, the NRC had outgrown its initial charges and it had become committed to projects that would make it, as an organization, too slow to react if a new war effort of a different sort was launched. The Council of National Defense, consisting of the Secretaries of War, Navy, Interior, Agriculture, Commerce and Labor, was authorized to create subordinate organizations for “support in special circumstances,” and so the *National Defense Research Committee* (NDRC) was established in 1940 with the approval of President Roosevelt ([4]). VANNEVAR BUSH (1890-1974), then President of the Carnegie Institution of Washington, was named Chairman of the committee by Roosevelt. A year later, Executive Order No. 8807 established the *Office of Scientific Research and Development* (OSRD), a reorganization of the NDRC, that emphasized carrying a research project from planning through the engineering stage. The OSRD also provided a place for the research programs of the Army, Navy, NDRC, and National Advisory Committee on Aeronautics (NACA) to discuss common goals. The OSRD maintained parallel committees on weapons and medicine.

In 1942, about a year after the declaration of war, the OSRD was reorganized to include a more transparent and correlated administrative structure. The five divisions of the NDRC with their manifold sections were split into 19 Divisions with more focussed charges. For example,

*The field covered by Division 7 included directors for airborne, land based and shipborne uses; servomechanisms for transmitting data to and from the guns and for elevating and traversing the guns; special range finders and testing equipment for assessing aerial gunnery and for evaluating the performance of directors, and the development of gun sights for special purposes.* ([4])

More general panels of experts were also organized under OSRD, including the *Applied Psychology Panel*, and the *Applied Mathematics Panel* (AMP), which was set up with the goal:

*... to aid NDRC divisions and Services by providing assistance in the handling of problems needing the special training of mathematicians.* ([Stewart])

WARREN WEAVER (1894-1978), then at the Rockefeller Foundation, was tapped out by Bush to head Section 7.5, *Fire Control Analysis*. Analytical studies in fire control and

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<sup>1</sup> Talk given at the joint international AMS-DMV-ÖMG meeting in Mainz, Germany, June 17–19, 2005.

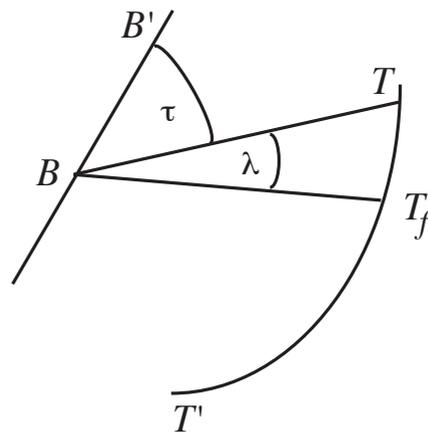
in other areas required more mathematical experts and that need was met by the AMP, also headed by Weaver.

The goal of this paper is to give an account of a particular mathematician, HASSLER WHITNEY (1907–1989), engaged in a particular problem, fire control for rockets (air-to-air), and the subsequent results of his efforts. Whitney is a more typical case, being a consultant and not an administrator. This account is based on research in the US National Archives where reports commissioned by the OSRD and associated documents are accessible. We will follow a problem from official *informal request* to the AMP, its assignment to a consultant, through the research, to delivery of results. From this we can get a sense of how mathematicians contributed to the war effort. I include a short account of some of the mathematics involved to illustrate the kind of analysis, and later exposition, that pure mathematicians produced for the AMP.

#### WHAT IS FIRE CONTROL?

The main problem of *fire control* is mechanical: an operator (called the *bomber*  $B$ ) controls both a sight and a gun. Since a projectile requires time to cover distance and the target (the *fighter*  $T$ ) is moving, it would be efficient if the mechanism for sighting included a means to calculate from input data, such as the *angular velocity* of the target, its *range* and the *time of flight* of the projectile, the direction that the gun must be pointed so that the projectile will meet its target. In simple realistic cases, this problem is susceptible to mathematical analysis which would suggest features in the design and use of equipment. The focus of Section 7.5 at Columbia's Applied Mathematics Group (AMG-C) was this analysis.

The prime case for improvements in fire control systems was data on anti-aircraft fire from the bombing of England. Weaver recalled, “. . . in October 1940 it was estimated that at least 10,000 rounds of fire from 3-inch anti-aircraft guns were expended for each plane shot down in the London area. Even that figure, moreover, is almost certainly too low.”



To understand the kind of analysis involved, consider the path of a fighter  $TT'$  relative to a bomber at  $B$ . Of course, the bomber can be moving, if defending from a plane or a ship, or not moving if on the ground at an anti-aircraft gun. It is the relative motion that is relevant. At time  $t$  the fighter is at a position  $T$  and the bomber wants to fire a shot to arrive at the fighter some time later, when the fighter is at  $T_f$ . The point  $T_f$  is

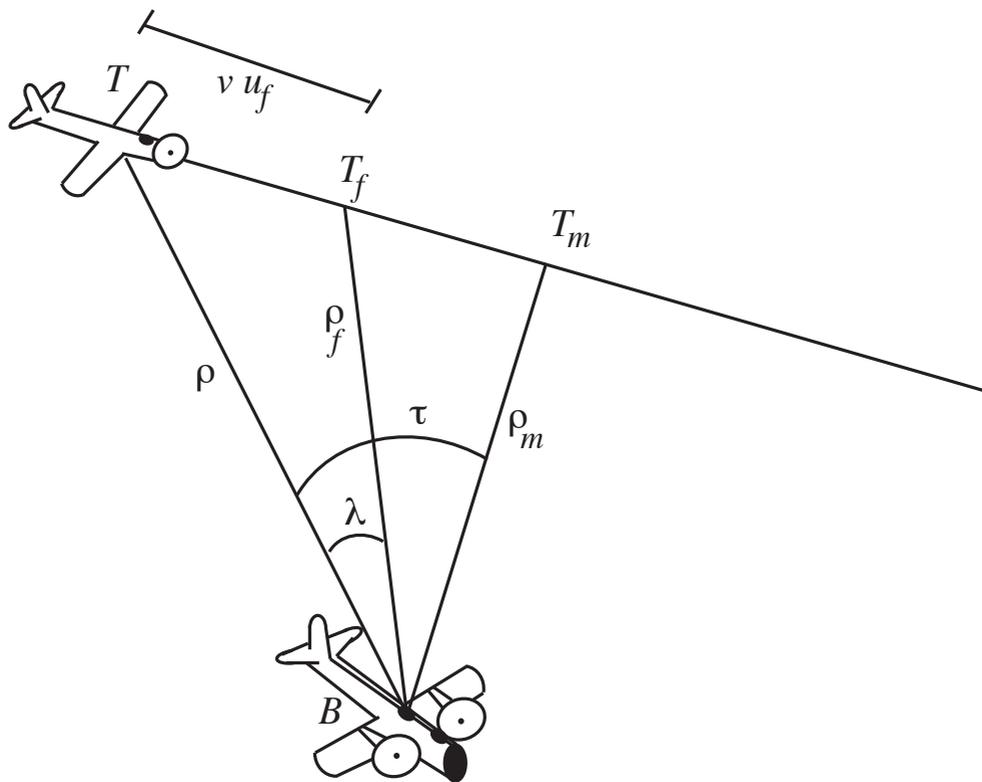
called the *future position* associated to  $T$ . The angle  $\lambda$ , called the **kinematic lead angle**, is the direction in which the shot should be fired to meet the fighter at  $T_f$ . It depends on features of the motion of the fighter, such as present *range* from the bomber (length of  $BT$ ), and its angular velocity, as well as the ballistic properties of the bomber's projectile.

Choose a reference line  $BB'$  from which to measure angles and view the path in polar coordinates:

$$\begin{aligned}\tau &= \angle B'BT \text{ angular position of the target.} \\ \rho &= BT \text{ present range of the target.} \\ u &= \text{time of flight of projectile to the present position } T. \\ u_f &= \text{time of flight of projectile to the future position } T_f \\ &= \text{time required for the target to go from } T \text{ to } T_f.\end{aligned}$$

A *fire control system* takes as input flight times of projectiles  $u$ , and target rates, such as  $d\tau/dt$  and/or  $d\rho/dt$  obtained by the operator through the apparatus and gives as output an approximation of the kinematic lead angle  $\lambda$ .

To give an example of the kind of analysis that is possible in this setting, let us assume the fighter is following a straight line path at constant speed relative to the bomber.



Let  $T_m$  denote the point along the flight path closest to the bomber, with  $BT_m = \rho_m$  and  $BT_m$  perpendicular to  $TT_m$ . The point  $T_m$  is called the *crossover position*. Let  $v$  denote

the constant velocity of the fighter. It follows that the distance  $TT_f = vu_f$  since  $u_f$  is the time of flight of the projectile from  $B$  to the future position. To study the kinematic lead angle, we can apply the law of sines:

$$\frac{\sin \lambda}{TT_f} = \frac{\sin(\angle BTT_m)}{\rho_f} = \frac{\cos \tau}{\rho_f}.$$

The last equality follows from the right triangle  $\triangle BT_mT$ . Thus

$$\sin \lambda = TT_f \frac{\cos \tau}{\rho_f} = \frac{vu_f \cos \tau}{\rho_f} = v \cos \tau \frac{u_f}{\rho_f}.$$

From  $\tan \tau = \frac{TT_m}{\rho_m}$ , we can take the derivative which obtains

$$\frac{d\tau}{dt} \sec^2 \tau = \frac{v}{\rho_m},$$

since  $\rho_m$  is constant and  $TT_m$  changes at rate  $v$ . Then  $d\tau/dt = (v/\rho_m) \cos^2 \tau$ . From the right triangle  $\triangle BT_mT$  we know that  $\cos \tau = \frac{\rho_m}{\rho}$ , and so  $d\tau/dt = \cos \tau \frac{\rho_m}{\rho} \cdot \frac{v}{\rho_m} = \frac{v \cos \tau}{\rho}$ . Substituting this equation into the expression for  $\sin \lambda$  we get

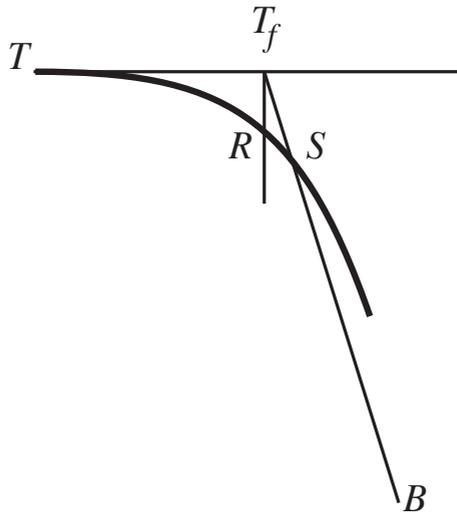
$$\sin \lambda = v \cos \tau \cdot \frac{u_f}{\rho_f} = \frac{v \cos \tau}{\rho} \cdot \frac{\rho u_f}{\rho_f} = \frac{u_f}{\rho_f} \cdot \rho \frac{d\tau}{dt}.$$

If we let  $W = \frac{\rho}{u}$  and  $W_f = \frac{\rho_f}{u_f}$  denote the *average velocities*, then

$$\sin \lambda = \left( \frac{W}{W_f} \right) u \frac{d\tau}{dt}.$$

For small angles  $\lambda$ , the value of  $\sin \lambda$  and  $\lambda$  are approximately the same, and for projectiles like rockets, average velocity is not so dependent on range and so  $W \sim 1$ . This motivates the choice of  $\lambda \sim u \cdot (d\tau/dt)$  for a good approximation, and calculable.

An application of this analysis appeared in the Memorandum [1] written by SAUNDERS MAC LANE (1909–2005) for AMG-C, *An Introduction to the Analytical Principles of Lead Computing Sights*. Compare the flight path of a fighter going on a straight line with one following a *pursuit curve*, that is, a curve of constant speed on which the fighter is always pointed at the bomber who is also following a flight path at a constant speed. In this case, the kinematic lead angle computed from the straight line path is ahead of the lead angle for the pursuit curve. However, the approximation  $\lambda = u\tau'$  gives the same value for both paths. To compensate for this situation, a bomber should take aim at a point behind the center of the fighter, a counterintuitive conclusion. Mac Lane described this consequence of the computation in his autobiography [2]:



*How should a machine gunner aim at an attacking fighter plane? The fighter is approaching the bomber, but the bomber is moving forward at the same time. The resulting rule is that the gunner should aim toward the tail, which is opposite from the rule for hunting ducks, where the rule for a stationary hunter is to aim ahead of the ducks. A major part of our problem was properly training machine gunners to aim toward the tail.*

A more complicated problem tackled by AMG-C is known as the *tracking problem*: what kind of design is best for the mechanism that controls a fire control system, that is, the device that translates the action of the bomber in tracking a target into the motion of the gun? Problems like this arrived at the desk of the AMP through *Informal Requests*, which were transmitted to the group from the armed forces through Section Chiefs, and then assigned to consultants who were university professors recruited and employed by the OSRD. One such request is the following:

**Informal Request #18: Analytical Study of the Tracking Problem**

Transmission: Dr. H. C. Wolfe. Section 7.2 (NDRC) at AMG-C

The extensive use of laboratory machines to study tracking problems, at the Franklin Institute and elsewhere, indicates the desirability of an analytical study of the nature of the tracking problem and of machines for tracking tests. The importance of such study has been recognized by Dr. Hassler Whitney (largely on his own initiative).

This request was assigned to Whitney and led to the AMP Note No. 21 entitled *Tracking and the fire control problem*.

#### HASSLER WHITNEY

Hassler Whitney studied music and physics at Yale University, graduating in 1929 and going on to Harvard University where he studied mathematics, obtaining a Ph.D. in 1932 under G.D. Birkhoff with a thesis on graph theory and the four-color problem. Already an instructor at Harvard from 1930, he became an assistant professor in 1932, associate in 1940, and full professor in 1946. He stayed there until 1952 when he became a permanent member at the Institute for Advanced Study in Princeton. During 1931-32 he visited Princeton as a National Research Council Fellow.

In the late 1930's, Saunders Mac Lane was also at Harvard as an assistant professor and this connection brought Whitney into the Applied Mathematics Group at Columbia.

He was employed as consultant to AMG-C officially 1.XI.43–30.IX.45 (also working 6.X.45–31.X.45 without pay) according to Sard’s final report on the AMG-C.

In a letter to Weaver a year into his consulting, Whitney expressed the kind of commitment he had for the project and his sense of what was needed to bring it to completion:

*“If the rocket and bombing sighting problems are really to be pushed (they must be!), it is essential that there be sufficient coordinated work on the problem. . . . With a good group to study all angles of the problem thoroughly, plenty of laboratory facilities for building, and planes and pilots available to carry out a (fairly large) variety of tests—tests to help both in the theoretical stage and in later stages—miracles could be accomplished, and quickly.”*

What Whitney had in mind was evident in remarks of MINA REES (1902–1997), technical aide and executive assistant with the AMP, who commented on Whitney in her 1980 article [3] on the war effort:

*That part of the program of the Applied Mathematics Panel that was concerned with the use of rockets in air warfare was primarily the responsibility of Hassler Whitney, who served as a member of the Applied Mathematics Group at Columbia. He not only integrated the work carried on at Columbia and Northwestern in the general field of fire control for airborne rockets but maintained effective liaison with the work of the Fire Control Division of NDRC in this field and with the activities of many Army and Navy establishments, particularly the Naval Ordnance Test Station at Inyorkern, the Dover Army Air Base, the Wright Field Armament Laboratory, the Naval Board of Ordnance, and the British Air Commission.*

Whitney apparently had a feel for this kind of work. Rees remarked elsewhere [8]:

*Hassler Whitney . . . “turned out to have an absolute genius for airplane problems from guidance studies.” Rees once accompanied Whitney to perform some tests on a training machine for pilots. Whitney was developing mathematical principles to discover the best techniques for aerial gunnery, and he was going to test his theories on a flight simulator. Rees and Whitney each got into a simulator, and “he shot down all his planes and I only shot down a couple. It was quite clear that he really knew all about this mechanism which I didn’t think he knew anything about. So a lot of people you didn’t really appreciate until you saw what they’d actually done.”*

#### WHITNEY’S ANALYSIS

In a fire control system much depends on the flow of information from the operator (the bomber) and the gun. If it is particularly difficult to move the sighting device to follow a target, then a computing device that determines the kinematic lead angle will deliver an out-of-date prediction while the mechanical turning takes place. A *lead computing sight* (lcs) incorporates the computing device and the sighting mechanism—when the operator sights the target in the cross-hairs, then the lcs has compensated for the kinematic lead angle and the gun is pointed to meet the target at the future position. The theory of such sights lies in the kind of tracking in use. Whitney assumed that the operator controlled a *handlebar*, a stick, “pivoted several feet in front of the operator’s chest, and pointing towards him; it may be moved up and down and to the right and left.” The handlebar controlled the movement of the sight, part of a fire control system which aimed the gun appropriately.

Three sorts of tracking were studied by the AMG-C:

**Direct tracking:** The position of the sight line depends only on the position of the handlebar.

**Velocity tracking:** The velocity of the sight line depends on the position of the handlebar.

**Aided tracking:** The position and velocity of the sight line depend on the position of the handlebar.

In direct tracking, the handlebar must be moved constantly to follow a moving target, while in velocity tracking, if the line between the operator and the target is turning at a fairly constant rate, then only small handlebar motions are required. However, this kind of tracking requires more skill and training, as Whitney writes *“Since the sight line motion is the integral of the handlebar motion, the tracking can hardly help but feel more complicated to the inexperienced operator.”*

Such practical concerns were part of the fabric of Whitney’s report. His analysis was written with the armed forces as audience, in war time when effort was meant to give concrete results, and by an eager researcher. He recommended certain systems for ease of use as a “psychological advantage.” He could imagine the bomber vividly:

*We remark, finally, that an important consideration in assessing a tracking mechanism is the mental effort it requires on the part of the operator. He may have other matters to attend to at the same time, such as ranging and pressing a trigger. In the heat of combat, one cannot expect a gunner to go through mental gymnastics.*

The analysis Whitney made was presented in general terms, incorporating as many features of a general fire control system as were useful. The basic variables he denoted by

$$\begin{aligned}\sigma(t) &= \text{sight direction,} \\ \gamma(t) &= \text{gun direction,} \\ \mu(t) &= \text{handlebar position,} \\ u &= \text{time of flight setting.}\end{aligned}$$

It follows that the lead kinematic angle  $\lambda$  satisfies  $\lambda = \gamma - \sigma$  and so our previous discussion applies. He assumed that  $\gamma$  and  $\sigma$  were determined by “linear differential equations between the above quantities and  $\mu$ , with coefficients depending on  $u$  only.” For example, aided tracking has an equation of the form

$$\frac{d\sigma}{dt} = A \frac{d\mu}{dt} + B\mu.$$

The basic differential equation governing the lead kinematic angle is derived in Mac Lane’s Memorandum [1]

$$cu \frac{d\mu}{dt} + \mu = u \frac{d\sigma}{dt}.$$

Equations of this sort resemble the equations for smoothing circuits, where the unknown function is a kind of smoothed version of the forcing function  $u\sigma'$ . They arise elsewhere in

electrical applications such as a resistance-capacitance circuit. The effect of smoothing is to take jerks in direction and exponentially dampen their effect on the system.

Whitney massaged the various classes of equations in a rather technical chapter leading to an analysis of *tracking gun error*, the difference between the ideal direction and the computed direction of the gun at firing. He ended with, “A general conclusion that may be drawn is that **any change in a turret mechanism designed to allow faster turret motions, thus improving tracking, could easily give worse gun positions.**” (Whitney’s emphasis.)

#### INYOKERN, CALIFORNIA

*July 18–20, 1945, Meeting of the Advisory Panel, Inyokern, California*

Whitney brought his analysis, written up in various reports that eventually led to his monograph AMP Note No. 21, to the Naval Ordnance Test Station (NOTS) about a month before the end of the war. He prepared a diary of the visit for the leadership of AMP. He was surprised on arrival to find that the working meeting he expected with top members of the Advisory Panel to research and development at NOTS had turned into a school to study the present situation in fire control systems and their assessment. He expressed his impatience in the diary [6]:

*Being somewhat perturbed at the program, and yet not being at all sure of what could be accomplished by such a large group of people with various degrees of specialization, HW did some lobbying to make the conference accomplish something. . . . However, HW did not feel himself in a position to take matters into his hands to any great extent, did not have much experience (if any) in this kind of situation, and was generally lazy and shy about talking too much.*

He saw the goal of this meeting to be to move forward the process to get a recommendation for an interim rocket sight. Although a schedule of talks had been laid out in advance, Whitney was added to the schedule, giving a talk, “*designed to help the members and spectators (thirty or more in all) fit the various sights into a pattern based on a simple analytical theory.*” Four sights were being tested at NOTS, and they were presented and demonstrated as part of the meeting. Although assessment was the goal, Whitney found the talks on assessment “dull,” while describing the talk by Comdr. Hayward in which he ranked the sights as the beginning of “fireworks.”

By the afternoon of the 20th, recommendations were drawn up based on the experience of pilots, and not the kind of analysis Whitney provided. He took the matter into his hands and got a smaller group together that evening. For this group, he took two sights that had been rated earlier as 2 and 3 in the list and analyzed them “from all angles” and arrived at the suggestion that their features be combined. In a surprise to Whitney, the ranking commanders decided “at once” that this would be the interim sighting system. Whitney’s efforts to ground the discussion, and his willingness to do some social maneuvering, calling special meetings, got an unexpected result.

#### SUMMARY

Whitney cannot have swayed a panel with purely technical arguments. His attendance to detail in his report, the diary, and in the descriptions by others of his involvement

present the picture of a committed contributor to the war effort, someone willing to go all the distance humanly necessary to make, in this case, improvements in weapon systems and bring the war to a close.

Other work on sights was successful. Weaver reported in his autobiography that French fliers were particularly happy with a sight for submarine bombing that was a product of analysis and development by OSRD.

In this study, we get a sense of the kind of activities that American mathematicians engaged in during World War II, of a mathematical nature, and of a social nature. Whitney, as subject, presents an interesting case of personal depth in pursuit of goals. This was probably typical. The work of the AMG-C was recognized for its contributions: it received the Naval Ordnance Development Award “for distinguished service to the research and development of Naval Ordnance” ([3]).

#### BIBLIOGRAPHY

- [1] Mac Lane, Saunders, An introduction to the analytical principles of lead computing sights, March 27, 1944, AMP Memo. 55.1, AMG-C 137, OSRD Report 4037.
- [2] Mac Lane, Saunders, A Mathematical Autobiography, A K Peters, Wellesley, MA, 2005.
- [3] Rees, Mina, The mathematical sciences and World War II, Amer. Math. Monthly, **87**(1980), 607–621.
- [4] Stewart, Irvin, Organizing Scientific Research for War: The Administrative History of the Office of Scientific Research and Development, Little, Brown and Co., Boston, 1948.
- [5] Weaver, Warren, Scene of Change: A Lifetime in American Science, Scribner’s, New York, 1970.
- [6] Whitney, Hassler, July Meeting of the Advisory Panel, Inyorkern: Diary of Hassler Whitney, July 25, 1945, AMG-C working paper No. 460.
- [7] Whitney, Hassler, Tracking and the fire control problem, September 14, 1945, AMP Note no. 21, AMG-C 441, OSRD Report 5680.
- [8] Williams, Kathleen Bloome, Improbable Warriors: Women Scientists and the U.S. Navy in World War II, Naval Institute Press, Annapolis, MD, 2001.