



# Quantum Superposition in Triple-Slit Diffraction Experiments

## Lesson Guide

# Objective

Understand the limitations of the superposition principle in analyzing slit diffraction experiments

# Lesson Tools

This lesson guide employs three primary tools in helping students reach the objective:

**Key concepts:** Take-home messages or ideas that reappear throughout the lesson

**Class prompts:** Questions posed to the class that probe students' knowledge or recall of relevant concepts

**Conceptual questions:** Questions that push students to understand the physics conceptually – beyond just the equations

# Review: Quantum Mechanics Fundamentals

In quantum mechanics, the state of a particle is described by a wavefunction, denoted by  $\Psi$ .

**Conceptual question:** What is the physical meaning of a wavefunction? Is it something an experiment can measure?

**Answer:** A wavefunction itself has no physical meaning – it cannot be measured directly.  $\Psi$  is just a solution to the **Schrodinger equation**, a differential equation which describes how a particle's wavefunction will evolve over time.\*

**Key concept:** The Schrodinger equation is analogous to Newton's law,  $F = ma$ .

Solving this equation of motion reveals the position and momentum of a physical system, which completely describe the system's state at a particular time. Analogously, solving the Schrodinger equation reveals the time evolution of a quantum system in the form of a wavefunction.\*

# Review: the Superposition Principle

One of the fundamental pillars of quantum mechanics, within the framework of the Schrodinger equation, is called the **superposition principle**

**Key concept:** the superposition principle requires that the state, or wavefunction, of a particle can be represented as a sum of substates:

$$\Psi_{AB} = \Psi_A + \Psi_B^*$$

This follows from the fact that  $\Psi$  is a solution to a second order linear partial differential equation (the Schrodinger equation), so any sum of solutions is also a valid solution

# Slit Diffraction Experiments

Applying the Superposition Principle

# Review: Young's Double-Slit Experiment

**Class prompt:** What is a slit diffraction experiment?

**Answer:** In 1803, the English Physicist Thomas Young first conducted the double-slit experiment. He found that even a single photon at a time fired through a pair of slits could interfere with itself and produce an interference pattern on a screen, demonstrating the wave-particle duality of light and the probabilistic nature of quantum mechanics. The double-slit experiment has since been recreated with varying numbers of slits.\*

**Key concept:** Slit diffraction experiments allow a photon to travel from the source to the screen along multiple unique trajectories. Each possible trajectory in a slit diffraction experiment can be assigned a unique wavefunction.

\*Eibenberger, Sandra; et al. (2013). "Matter-wave interference with particles selected from a molecular library with masses exceeding 10000 amu". Physical Chemistry Chemical Physics. 15 (35). doi: 10.1039/C3CP51500A

# The Superposition Principle in Slit Diffraction Experiments

Suppose a particle passes through a diffraction grating with two slits, A and B, which can be either open or closed

**Conceptual question:** Can we apply the superposition principle to a particle in a double-slit experiment? If the particle is in the state  $\Psi_A$  when only slit A is open, and  $\Psi_B$  when only B is open, then with both slits open, is it in the state  $\Psi_A + \Psi_B$ ?



**Answer:** No, the wave function with both slits open differs slightly from the sum of the wave functions for each slit open individually.

**Key concept:**  $\Psi_A$  and  $\Psi_B$  satisfy different **boundary conditions**, while the superposition principle only applies to states with shared boundary conditions. So, in a double slit experiment,

$$\Psi_{AB} \neq \Psi_A + \Psi_B.*$$

\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.

# Non-Classical Paths

The discrepancy between  $\Psi_{AB}$  and  $\Psi_A + \Psi_B$  is due to a contribution from **non-classical paths** (NC paths).

**Key concept:** NC paths are trajectories that do not pass directly from the source through one slit to detector, but rather loop or kink somewhere in the middle such that they cross the plane of the slits more than once.

# Verifying the Effect of NC Paths on the Wavefunction

First consider a double-slit experiment consisting of a photon source, a diffraction grating with two slits – A and B – and a detection screen

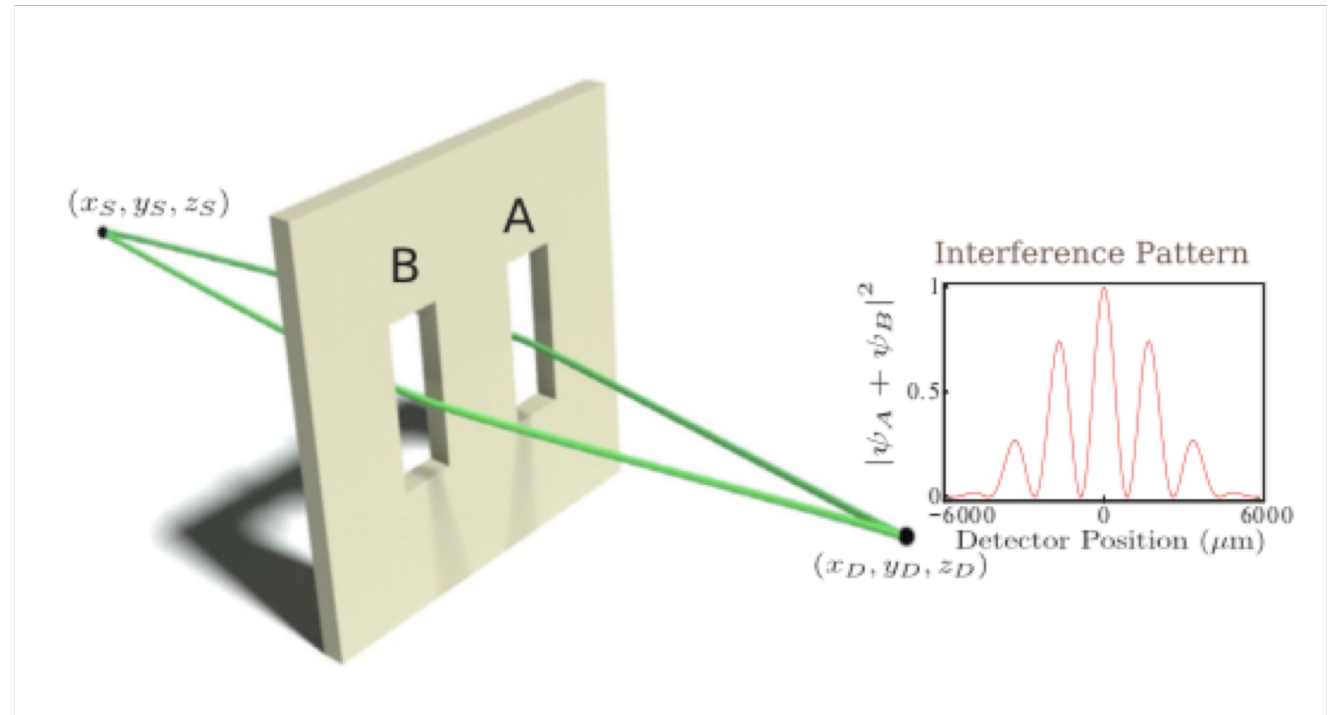


Figure I. Double-slit experiment showing the interference pattern obtained by assuming  $\Psi_{AB} = \Psi_A + \Psi_B^*$

\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.

Ignoring NC paths the wavefunction would be

$$\Psi_{AB} = \Psi_A + \Psi_B^* \quad (1)$$

**Class prompt:** Assuming (1) is true, what is an expression for the probability of detecting a particle at some position on the screen?

**Answer:** Detection probability is related to the wavefunction by  $P = |\Psi|^2$ . This is **Born's postulate**. Applying Born's postulate to (1) gives

$$P_{AB} = |\Psi_{AB}|^2 = |\Psi_A + \Psi_B|^2 ** \quad (2)$$

Expanding the right side of (1) gives

$$P_{AB} = |\Psi_A|^2 + |\Psi_B|^2 + I_{AB} = P_A + P_B + I_{AB} ** \quad (3)$$

where

$$I_{AB} = \Psi_A^* \Psi_B + \Psi_A \Psi_B^* **$$

\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi: 10.1103/physrevlett.113.120406.

\*\*Barnea, R. A., Cheshnovsky, O., & Even, U. (2017). Matter-waves approaching limits predicted by Feynman path integrals for multipath interference. *Physical Review Letters*, 97(2). <https://doi.org/10.1103/PhysRevA.97.023601>

**Key concept:**  $I_{AB}$  is a term due to interference between the two wavefunctions. It is the difference between the probability for both slits open and the sum of the probabilities for each slit open individually:

$$I_{AB} = P_{AB} - (P_A + P_B)^* \quad (4)$$

# Triple-Slit Diffraction

Now consider a diffraction grating with three slits - A, B and C – in which the effects of NC paths become more pronounced.

Ignoring NC paths, the wavefunction would be  $\Psi_{ABC} = \Psi_A + \Psi_B + \Psi_C$

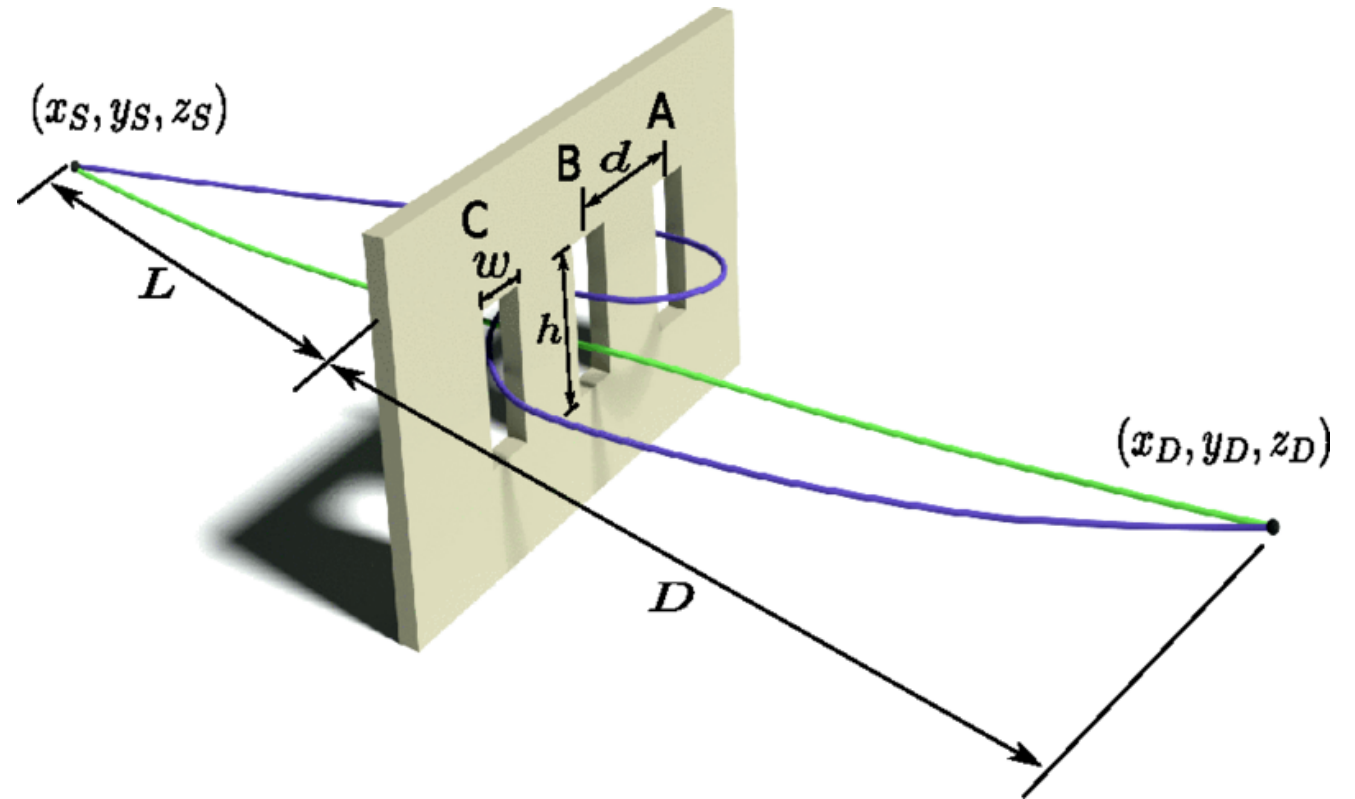


Figure 2. Triple-slit experiment with the green line representing a classical path and the purple curve representing a NC path\*

\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.

Just as in the double-slit experiment, interference will occur between each pair of slits. But now there are three different pairs of slits, so the probability includes three interference terms:\*

$$P_{ABC} = P_A + P_B + P_C + I_{AB} + I_{BC} + I_{AC}^* \quad (5)$$

It follows from (4) that  $I_{BC} = P_{BC} - (P_B + P_C)$  and  $I_{AB} = P_{AC} - (P_A + P_C)$ .\*

Plugging the interference terms into (5) gives

$$\begin{aligned} P_{ABC} &= P_A + P_B + P_C + P_{AB} - P_A - P_B + P_{BC} - P_B - P_C + P_{AC} - P_A - P_C \\ &= P_{AB} + P_{BC} + P_{AC} - P_A - P_B - P_C^* \end{aligned} \quad (6)$$

\*Sinha, U., et al. Ruling Out Multi-Order Interference in Quantum Mechanics. (2010). *Science*, 329(5990), pp. 418–421., doi:10.1126/science.1190545.

\*\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.

\*\*\*Eibenberger, Sandra; et al. (2013). "Matter-wave interference with particles selected from a molecular library with masses exceeding 10000 amu". *Physical Chemistry Chemical Physics*. 15 (35). doi: 10.1039/C3CP51500A

Collecting all the terms on the left hand side of (6) gives

$$P_{ABC} - P_{AB} - P_{BC} - P_{AC} + P_A + P_B + P_C = 0^{**} \quad (7)$$

The left hand side of (7) is called the **Sorkin parameter**, denoted by  $\epsilon$ .\*\*\*

Equation (7) shows that  $\epsilon = 0$  when NC paths are ignored. Confirming the presence of NC paths in triple-slit experiments requires computing the Sorkin parameter accounting for NC paths and showing that it is non-zero.\*

\*\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.



# Computing the Sorkin Parameter

The Sorkin parameter can be written in terms of functions called **free particle propagators**, denoted by  $K$ . Propagators give the probability amplitude associated with a particle travelling between two points along a particular path\* - in this case, a photon travelling from the source through to the detection screen passing through at least one slit. For a particle travelling from an arbitrary position  $\mathbf{r}$  to a final position  $\mathbf{r}'$ , the propagator function has the form

$$K(\mathbf{r}, \mathbf{r}') \propto \frac{1}{|\mathbf{r} - \mathbf{r}'|} e^{ik|\mathbf{r} - \mathbf{r}'|} **$$

\*The mathematics of PDEs and the wave equation, p 32., Michael P. Lamoureux, University of Calgary, Seismic Imaging Summer School, August 7–11, 2006, Calgary.

\*\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.

Because absolute probability is defined to be the magnitude squared of the probability amplitude, the Sorkin parameter can be written in terms of the propagators for the different slit configurations:

$$\epsilon = |K_{ABC}|^2 - |K_{AB}|^2 - |K_{BC}|^2 - |K_{AC}|^2 + |K_A|^2 + |K_B|^2 + |K_C|^2^*$$

Each of these propagators is composed of two terms, representing the classical and NC paths:

$$K = K_C + K_{NC}^*$$

$K_C$  and  $K_{NC}$  need to be computed for each slit configuration, which involves integrating  $K$  over all possible paths. This is called a **path integral**. In a given slit configuration, the classical paths are constrained by the areas of the “open” slits, so those areas serve as the domains of integration in computing the  $K_C$ ’s. NC paths, however can pass through “open” and “closed” slits, so computing the  $K_{NC}$ ’s requires a second integration over the areas of the “closed” slits.\*

\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.

# Results of the Sorkin Parameter Computation

Using the path integral method, the Sorkin parameter turns out to be non-zero – on the order of  $10^{-6}$  at the central maximum of the interference pattern for 810 nm photons – confirming the presence of NC paths\*,\*\*

**Key concept:** Because of the contribution from NC paths, the wavefunction of a particle in a triple-slit experiment is not a superposition of the wavefunctions for the three individual slits.

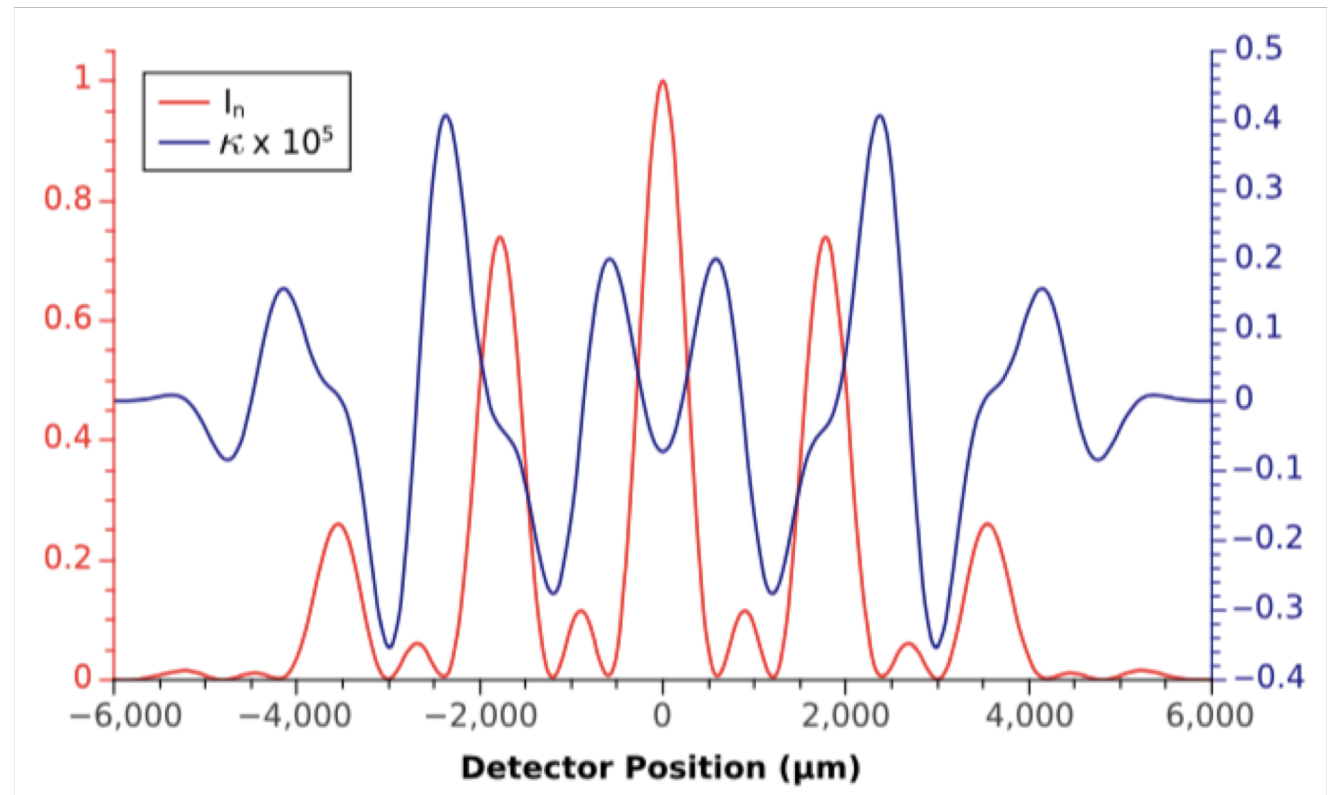


Figure 3. The blue curve represents  $\kappa$  as a function of position on the screen, where  $\kappa$  is a calculated quantity proportional to  $\epsilon$ . The red curve is the triple-slit interference pattern.\*

\*Sawant, Rahul, et al. (2014) Nonclassical Paths in Quantum Interference Experiments. *Physical Review Letters*, 113(12), doi:10.1103/physrevlett.113.120406.

\*\*Sinha, U., et al. Ruling Out Multi-Order Interference in Quantum Mechanics. (2010). *Science*, 329(5990), pp. 418–421., doi:10.1126/science.1190545.

# Review of Key Concepts

1. The Schrodinger equation is analogous to Newton's law,  $\mathbf{F} = m\mathbf{a}$ . It serves as an equation of motion for a quantum system.
2. The superposition principle requires that the state, or wavefunction, of a particle can be represented as a sum of substates.
3. Slit diffraction experiments allow a photon to travel from the source to the screen along multiple unique trajectories. Each possible trajectory in a slit diffraction experiment can be assigned a unique wavefunction.
4. In a double slit experiment,  $\Psi_{AB} \neq \Psi_A + \Psi_B$  because  $\Psi_A$  and  $\Psi_B$  satisfy different boundary conditions, while the superposition principle only applies to states with shared boundary conditions.
5. NC paths are trajectories that do not pass directly from the source to detector, through one slit, but rather loop or kink somewhere in the middle such that they cross the plane of the slits more than once.
6. Interference occurs between every pair of slits. The detection probability for both slits open is not equal to the sum of the probabilities for each slit open individually, resulting in an interference term for each pair of slits.
7. Because of the contribution from NC paths, the wavefunction of a particle in a triple-slit experiment is not a superposition of the wavefunctions for the three individual slits.