Almost alternating links and the Jones polynomial

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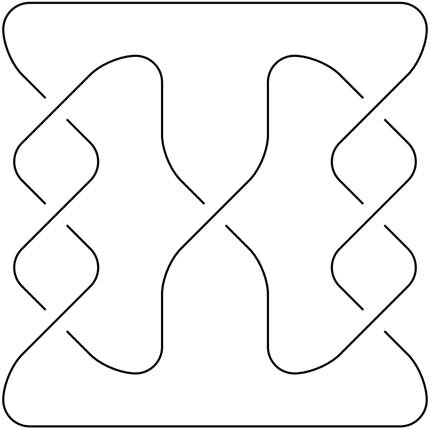
Alternating knots

A knot diagram is *alternating* if the crossings alternate under, over, under, over ... as one travels along the knot. A knot is called *alternating* if it has an alternating diagram.

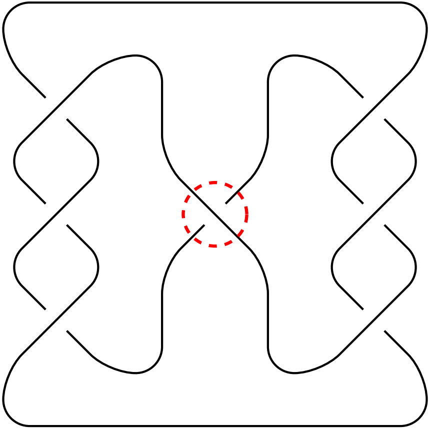


Properties of alternating knots

- An alternating diagram of K determines the crossing number of K.
- From an alternating diagram, we can determine whether K is prime.
- The complement of an alternating knot has a "nice" geometric structure.
- Many knot invariants are "easy" for alternating knots.

Almost alternating knots

A knot diagram is *almost alternating* if it can be transformed into an alternating diagram via one crossing change. A knot is called *almost alternating* if it is non-alternating and has an almost alternating diagram.



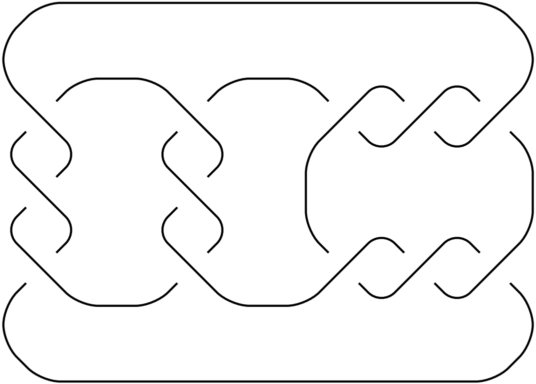
Examples of almost alternating knots

- All non-alternating knots with 10 or fewer crossings.
- All non-alternating knots with 11 crossings except possibly two: $11n_{95}$ and $11n_{118}$.
- All non-alternating pretzel knots on arbitrarily many strands (Kim, Lee).
- All Montesinos knots (Abe, Jong, Kishimoto).

Facts about almost alternating links

- Defined in 1992 by Colin Adams and seven undergraduate students.
- The complement of an almost alternating knot has a “nice” geometric structure.
- The Khovanov and knot Floer homologies of almost alternating knots are relatively simple.

Strategies for finding almost alternating diagrams



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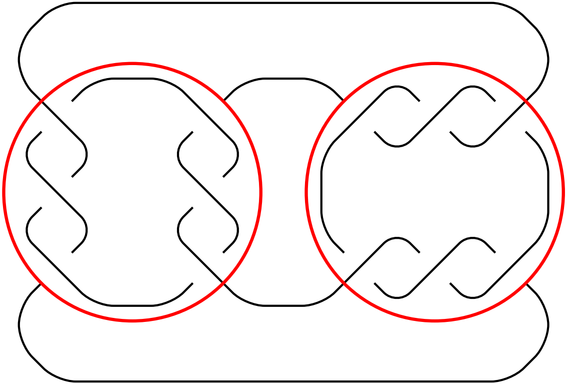

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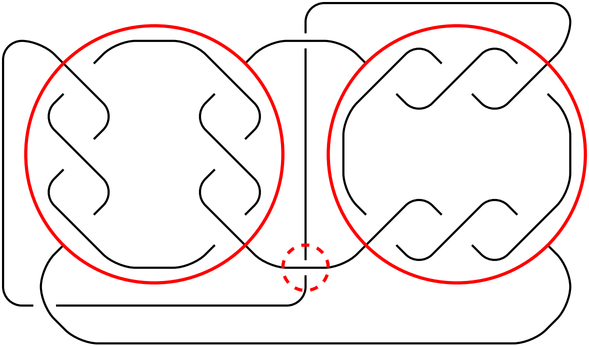
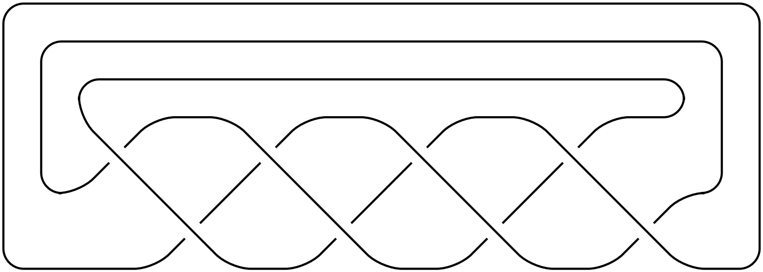


Another example: $T_{3,4}$



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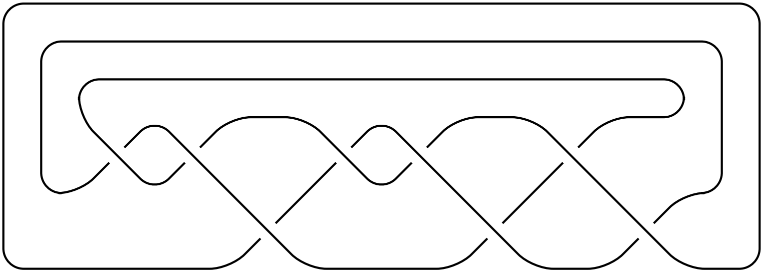

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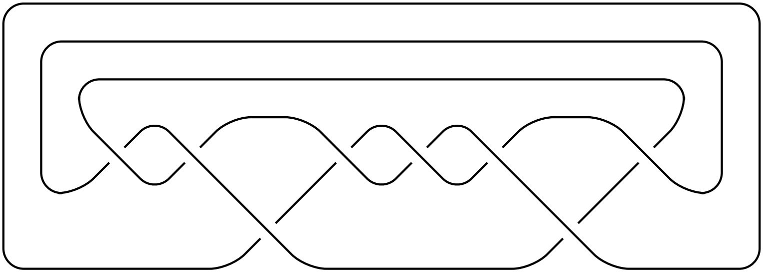

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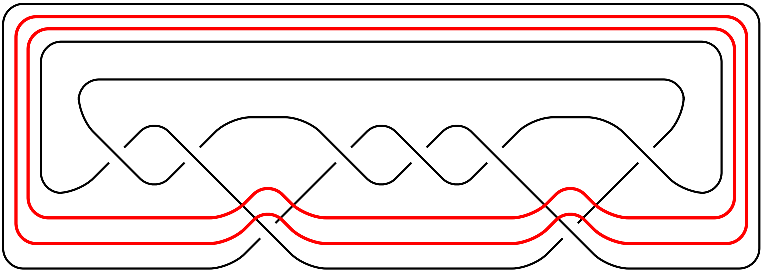
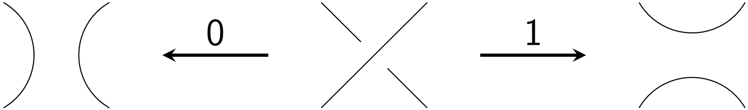


Not almost alternating?

Question: How can we determine when a non-alternating knot is not almost alternating?

One Answer: Use the Jones polynomial of the knot.

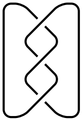
Kauffman state



Each crossing has an 0-resolution and a 1-resolution.

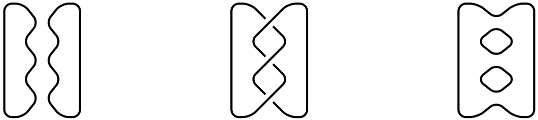
The collection of curves obtained by picking a resolution for each crossing is a *Kauffman state*.

The trefoil

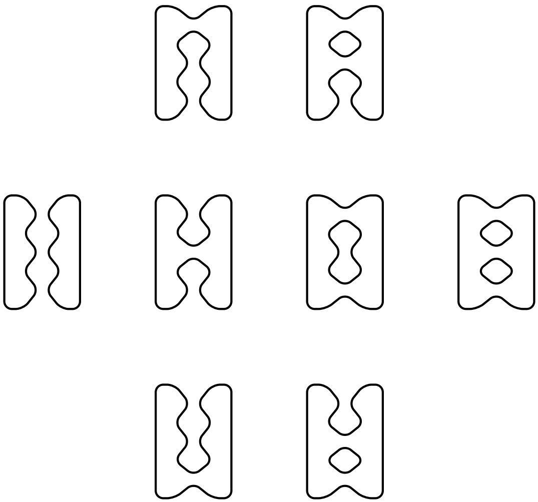


The all-0 state

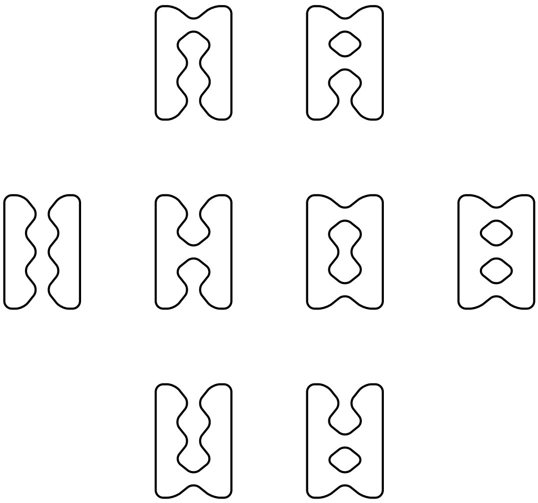
The all-1 state



All of the Kauffman states

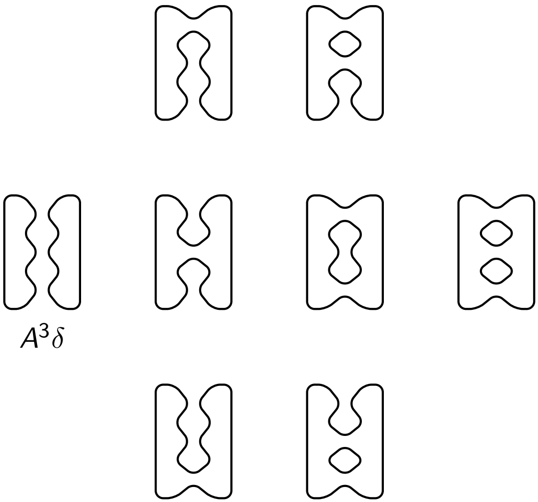


A polynomial to each state



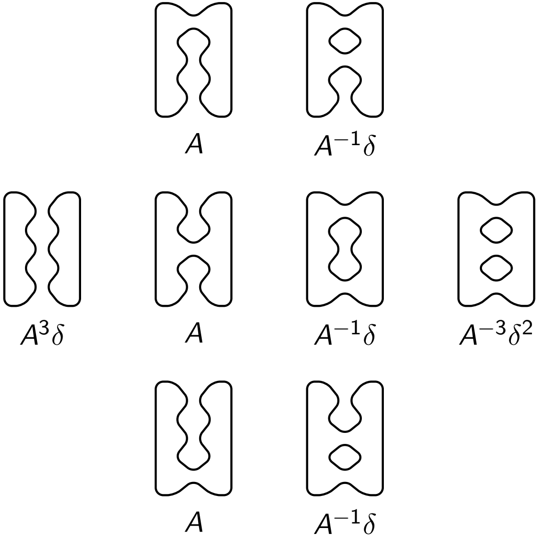
$A^{\#(0)-\#(1)}\delta^{\#(\text{circles})-1}$ where $\delta = -A^2 - A^{-2}$

A polynomial to each state



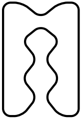
$$A^{\#(0)-\#(1)}\delta^{\#(\text{circles})-1} \text{ where } \delta = -A^2 - A^{-2}$$

A polynomial to each state

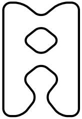


$A^{\#(0)-\#(1)}\delta^{\#(\text{circles})-1}$ where $\delta = -A^2 - A^{-2}$

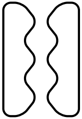
Add everything together



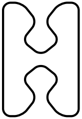
A



$A^{-1}\delta$



$A^3\delta$



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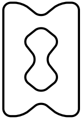
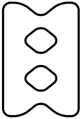
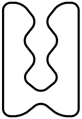


$A^{-1}\delta$



$A^{-3}\delta^2$



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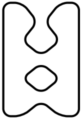
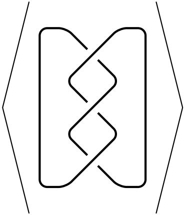


$A^{-1}\delta$

$$A^3\delta + 3A + 3A^{-1}\delta + A^{-1}\delta^2 = A^{-7} - A^{-3} - A^5$$

The Kauffman bracket and the Jones polynomial



$$= A^{-7} - A^{-3} - A^{5}$$

Jones polynomial $J(K,t) = (-A)^{-3w} \langle K \rangle |_{A = t^{-1/4}}$.

In our example $J(K,t) = t + t^3 - t^4$.

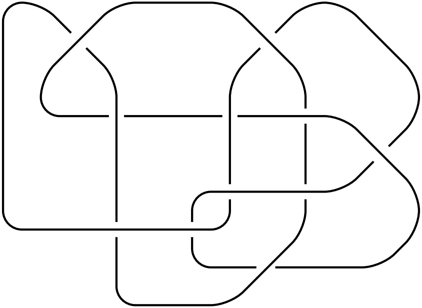
The Jones polynomial of an almost alternating knot

Let $J(K,t) = a_m t^m + a_{m+1} t^{m+1} + \cdots + a_{n-1} t^{n-1} + a_n t^n$ be the Jones polynomial of K where a_m and a_n are nonzero.

Theorem (Dasbach, L.)

If K is almost alternating, then either $|a_m| = 1$ or $|a_n| = 1$ (or both equal 1).

The knot $11n_{95}$



$$J(11n_{95}, t) = 2t^2 - 3t^3 + 5t^4 - 6t^5 + 6t^6 - 5t^7 + 4t^8 - 2t^9.$$

Our theorem implies $11n_{95}$ is not almost alternating.

Thank you!